

UNCLASSIFIED

AD NUMBER

ADB002555

LIMITATION CHANGES

TO:

Approved for public release; distribution is unlimited.

FROM:

Distribution authorized to U.S. Gov't. agencies only; Test and Evaluation; MAR 1975. Other requests shall be referred to Naval Surface Weapons Center, Dahlgren Laboratory, Dahlgren, VA 22448.

AUTHORITY

USNSWC ltr, 11 Mar 1976

THIS PAGE IS UNCLASSIFIED

THIS REPORT HAS BEEN DELIMITED
AND CLEARED FOR PUBLIC RELEASE
UNDER DOD DIRECTIVE 5200.20 AND
NO RESTRICTIONS ARE IMPOSED UPON
ITS USE AND DISCLOSURE.

DISTRIBUTION STATEMENT A

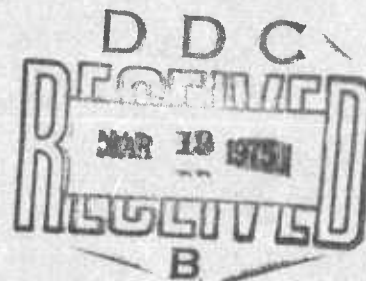
APPROVED FOR PUBLIC RELEASE;
DISTRIBUTION UNLIMITED.

NSWC/DL TECHNICAL REPORT TR-3082
March 1975

AD B002555

A STATIC STRUCTURAL ANALYSIS OF VARIOUS MAJOR COMPONENT JOINING DESIGNS FOR THE 8-INCH GUIDED PROJECTILE

Lawrence A. Mason



U.S. NAVAL WEAPONS LABORATORY
DAHLGREN, VIRGINIA



Distribution limited to U.S. Gov't. agencies only; Test and Evaluation (03-75). Other requests for this document must be referred to OIC and Asst. Commander, Naval Surface Weapons Center Dahlgren Laboratory, Dahlgren, Va. 22448.

This TECHNICAL NOTE is intended for intra-laboratory distribution only. It does not necessarily express the official views or judgment of the Naval Surface Weapons Center.

**NAVAL SURFACE WEAPONS CENTER
DAHLGREN LABORATORY
Dahlgren, Virginia
22448**

C. J. Rorie, Capt., USN
OIC, and Assistant Commander

Dr. J. S. di Rende
Deputy Technical Director

NSWC/DL Technical Report No. TR-3082
March 1975

**A STATIC STRUCTURAL ANALYSIS OF
VARIOUS MAJOR COMPONENT JOINING DESIGNS FOR
THE 8-INCH GUIDED PROJECTILE**

Lawrence A. Mason
Surface Warfare Department

Distribution limited to U. S. Government agencies only: Test and Evaluation; March 1975. Other requests for this document must be referred to OIC and Assistant Commander, Naval Surface Weapons Center, Dahlgren Laboratory, Dahlgren, Virginia 22448.

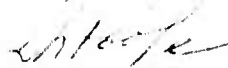
FOREWORD

The work described herein was accomplished under NAVORDTASK 035D-001/090-1/UF/32-343-501. An investigation of the structural analysis of several candidate joining designs for major components of the 8-Inch Guided Projectile was conducted.

This report was reviewed and approved by:

Thomas F. Doi, Senior Joint Design Engineer, Guided Projectile Division
Conrad W. Brandts, Technical Manager, 5-Inch Guided Projectile Program,
Guided Projectile Division
H. C. Oliver, Head, Guided Projectile Division

Released by


J. L. POOLE

Assistant Head
Military Requirements
Surface Warfare Department

ABSTRACT

This report contains a description of the structural analysis of several candidate joining designs for major components of the 8-Inch Guided Projectile.

The analyses contained herein are not intended to be rigorous in approach, but are conducted in such a manner as to cover the pertinent design details which would effect the overall structural integrity of both the joining mechanism itself and the adjacent sections of the major components involved.

The information contained in this report includes analyses of: (1) the three-section ring or "Marman" band concept for both the warhead/afterbody and warhead/guidance and control interfaces, (2) the two-ring inverted Marman band, (3) the press-fit approach (also for both warhead interfaces), and (4) the bolted-joint concept.

The results of these analyses indicate that the simpler bolted-joint design exhibits the greatest potential for success, although no design is completely void of potential problem areas. Recommendations are made to improve the design where possible.

CONTENTS

| | Page |
|--|------|
| FOREWORD | i |
| ABSTRACT | ii |
| LIST OF ILLUSTRATIONS | iv |
| LIST OF SYMBOLS | v |
| I. INTRODUCTION | 1 |
| II. THEORY AND DISCUSSION | 3 |
| III. RESULTS AND CONCLUSIONS | 14 |
| REFERENCES | 19 |
| APPENDICES | |
| A. Structural Analysis of the Bolted Three-Section Ring Afterbody/Warhead Interface Design | |
| B. Structural Analysis of the Bolted Three-Section Ring Warhead/Guidance and Control Interface Design | |
| C. Structural Analysis of the Bolted Two-Section Inverted Ring Design | |
| D. Structural Analysis of the Press-Fit Joint Designs | |
| E. Structural Analysis of the Four-Bolt Joint Design | |
| F. Distribution | |

LIST OF ILLUSTRATIONS

| Figure | | Page |
|--------|--|------|
| 1 | Eight-Inch Guided Projectile | 2 |
| 2 | Experimental Shock Pressure Pulses | 6 |
| 3 | Strength Comparison of the Afterbody/Warhead Interface Joint Designs | 16 |
| 4 | Strength Comparison of the Warhead/Guidance and Control Interface Joint Designs | 17 |
| A-1 | Three-Section Ring Joint for the Afterbody/Warhead Interface | A-2 |
| A-2 | Cross Section of the Band at the Bolt Stand | A-8 |
| A-3 | Cross Section of Joined Bodies at Joint | A-10 |
| B-1 | Three-Section Ring – Warhead/G&C Design | B-1 |
| B-2 | Three-Section Ring Stress Areas | B-3 |
| C-1 | Perspective View Inverted Ring Band – Warhead/Afterbody Design | C-1 |
| C-2 | Cross Section of Ring Band for Inverted Design | C-4 |
| C-3 | Cross Section of Inverted Design | C-6 |
| C-4 | Cross Section of Joined Bodies Under Ring | C-7 |
| C-5 | Dimensions for Thermal Considerations | C-10 |
| D-1 | Press Fit for the Warhead/G&C Interface Design | D-2 |
| D-2 | Press Fit for the Warhead/Afterbody Interface Design | D-3 |
| D-3 | Concentric Cylinders in a Press-Fit Analysis | D-4 |
| E-1 | Four-Bolt Joint Design | E-1 |

LIST OF SYMBOLS

| Symbol | Definition | Unit |
|-------------------------------|--|---------------------|
| A | Gun-bore cross-sectional area | in. ² |
| $A_a, A_b, A_c, \text{ etc.}$ | Cross-sectional areas at critical points | in. ² |
| A' | Area of projectile cross section | in. ² |
| a | Acceleration, linear | ft/sec ² |
| a_s | Sideload acceleration due to balloting | ft/sec ² |
| c | Average effective moment arm | in. |
| D | Gun-bore diameter | in. |
| D_i | Inside diameter of shell | in. |
| D_o | Outside diameter of shell | in. |
| E | Modulus of elasticity | lb/in. ² |
| F | Force | lb |
| F_b | Force in the bolts | lb |
| F_m | Total bending moment force | lb |
| F_o | Frequency excitation force, sideload | lb |
| F_p | Force due to chamber pressure | lb |
| F_R | Force due to reverse acceleration or rebound | lb |
| F_S | Force due to forward axial acceleration or setback | lb |
| F_T | Tangential shear force | lb |

LIST OF SYMBOLS (Continued)

| Symbol | Definition | Unit |
|-----------------------------|--|---------------------|
| $F_t, F_{t_b}, \text{etc.}$ | Tensile force in the bolts | lb |
| F_w | Force in the projectile wall due to rotation | lb |
| F_i | Radial force as a result of P_i | lb |
| g | Acceleration due to gravity | ft/sec ² |
| I | Area moment of inertia | in. ⁴ |
| I_p | Polar moment of inertia of metal parts forward of section considered | lb-in. ² |
| K | Stiffness constant | lb/in. |
| ℓ | Projectile length | in. |
| $L, L_b, \text{etc.}$ | Longitudinal length of section under consideration | in. |
| M | Bending moment due to gravity | lb-in. |
| M_x | Total bending moment due to sideload acceleration | lb-in. |
| m | Mass | lb _m |
| N | Number of knurling teeth | |
| n | Rifling twist in calibers per turn | |
| P | Breech-chamber pressure | lb/in. ² |
| P_i | Internal pressure on a cylinder | lb/in. ² |
| P_o | External pressure on a cylinder | lb/in. ² |

LIST OF SYMBOLS (Continued)

| Symbol | Definition | Unit |
|-------------------------------|---|---------------------|
| R | Radius | in. |
| R_c | Contact radius of the pressed-fit cylinders | in. |
| R_h | Radius to highest contact point of band | in. |
| R_i | Inside radius of the shell | in. |
| R_ℓ | Radius to lowest contact point of band | in. |
| R_o | Outside radius of the shell | in. |
| r | Any arbitrary radius | in. |
| SF | Safety factor | |
| S_s | Shear stresses | lb/in. ² |
| S_{1t} | Tensile stress in the band rings | lb/in. ² |
| S_1 | Total longitudinal or principle stress | lb/in. ² |
| S_{11} | Sideload stress | lb/in. ² |
| $S_{1b}, S_{1r}, \text{ etc}$ | Longitudinal stresses | lb/in. ² |
| S_2 | Total hoop or tangential stresses | lb/in. ² |
| $S_{2m}, S_{2n}, \text{ etc}$ | Hoop stresses | lb/in. ² |
| S_3 | Radial stresses | lb/in. ² |
| T | Temperature | Deg. F |
| τ | Time | sec |
| t_m | Thickness of the band bearing area under bolt | in. |

LIST OF SYMBOLS (Continued)

| Symbol | Definition | Unit |
|------------|--|---------------------|
| t_1 | Wall thickness of the shell | in. |
| V | Linear velocity of the projectile | ft/sec |
| W | Total projectile weight | lb |
| W_B | Weight of joining band | lb |
| W_B' | Weight of the rearward portion of the band | lb |
| W' | Weight of metal parts forward of section | lb |
| W'' | Weight of metal parts rearward of section | lb |
| x | Beam deflection | in. |
| α | Frequency damping factor, coefficient of expansion | in./deg |
| δ | Total displacement | in. |
| ζ | Frequency damping coefficient | |
| μ | Poisson's constant | |
| ρ | Density of materials | lb/in. ³ |
| σ_s | Shearing strength of the material | lb/in. ² |
| σ_y | Tensile yield strength | lb/in. ² |
| Ω | Vibrating frequency | cycle/sec |
| Ω_n | Natural vibrating frequency | cycle/sec |

LIST OF SYMBOLS (Continued)

| Symbol | Definition | Unit |
|------------|--------------------------------|---------|
| ω | Angular velocity | rad/sec |
| ω_B | Rotating band angular velocity | rad/sec |
| ω_p | Projectile angular velocity | rad/sec |

I. INTRODUCTION

The Surface Warfare Department was tasked as part of ORDTASK 035D-001/090-1/UF/32-343-501 (Eight-Inch Guided Projectile Program) to conduct a detailed structural analysis of several candidate projectile interface designs. Figure 1 shows the relationship of the various sections of the projectile. The interface designs considered included: (1) the three-ring "Marman" band concept for both the warhead/afterbody, (2) warhead/guidance and control interfaces, (3) the two-ring inverted Marman band, (4) press-fit designs for both interfaces, and (5) a four-bolt design. Structural analysis for individual candidate designs is located in Appendices A through E.

This effort consisted of a static analysis of each interface, including all significant stresses resulting from a gun-launch environment which would effect the structural integrity of the projectile, including both inertial and thermal stresses. In each instance, analysis followed four phases: (1) the determination of the maximum forces acting on the projectile body during gunfiring, (2) the determination, for critical points in the design, of the principle stresses resulting from these forces, (3) the utilization of yield criteria to determine if the stress at these critical points results in the material being in the elastic or plastic state, and (4) the determination of the effects of large temperature variations. Failure was assumed to occur when the stress at a particular critical point was sufficient to put the material in the plastic state of deformation.

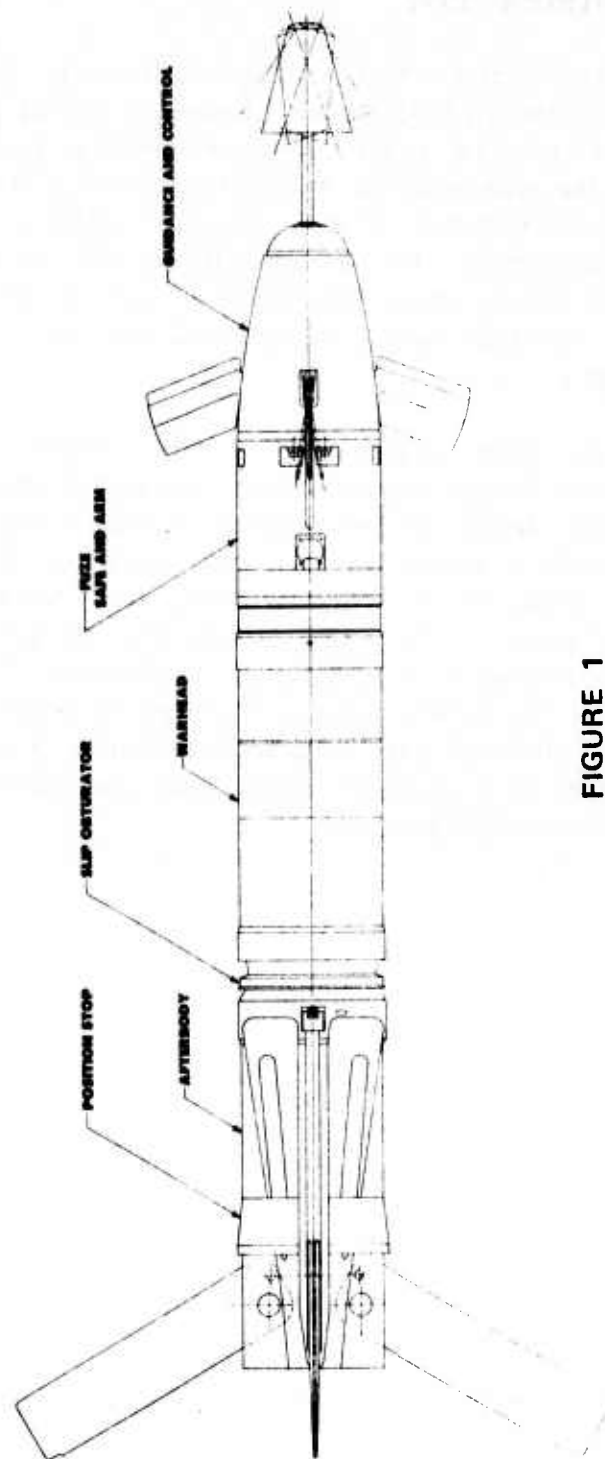


FIGURE 1
Eight-Inch Guided Projectile

II. THEORY AND DISCUSSION

Due to the characteristics of gunfiring, the resultant forces may be divided into three areas: (1) interior ballistic — initial acceleration forces, (2) interior ballistic — quasi-static forces, and (3) exterior ballistic forces. The "rebound" force experienced by a projectile upon exit from the gun muzzle is considered an interior ballistic force. Exterior ballistic forces were considered small relative to those applied by the pressure of the propellant gases treated here.

Normally, the forces occurring during gunfiring, which should be considered, include the following: (1) a force on the base and sides of the projectile rearward of the rotating band, and on the rear of the rotating band itself, due to the pressure of the propellant gases, (2) the inertial (setback and rebound) force in the projectile walls due to the acceleration or deceleration of the projectile, (3) the tangential force due to an angular acceleration imparted by the rifling, (4) a radial force due to spin, and (5) a radially compressive force resulting from the engraving of the rotating band.

Upon examining the 8-Inch Guided Projectile (GP) configuration, one can derive these basic conclusions. First, the rotating band employed on the projectile is actually a slip obturator. The radial compressive force exerted on the band seat when the band is being engraved will be negligible due to the low-yield strength of the band material. Secondly, both the rotating band and the forward bourrelet are located on the warhead section. Thus, the rear interface between the warhead and afterbody will be exposed to the full blunt of the pressure of the propellant gases.

The compressive setback force at any section of the projectile forward of the rotating band can be readily calculated by the simple equation

$$F_s = \frac{W'}{g} a \quad (1)$$

where W' is the weight of the metal parts forward of the section under consideration and a is the common acceleration of all parts. Use of this equation does, however, involve the assumption that the projectile acts as a rigid body, i.e., the internal energy stored is negligible.

For sections rearward of the rotating band, the compressive force is related to the chamber pressure, P , such that

$$F_p = PA' \quad (2)$$

where A' is the largest cross-sectional area of any section rearward of (and including) the section under consideration.

Spin will be imparted to the projectile by the rotating band through a force applied by the rifling twist. For a non-slip rotating band, this tangential force is transmitted totally to the body of the projectile and is related to the linear acceleration through the relation (Reference 1):

$$F_T = \frac{8I_p'}{n(D_o^2 + D_i^2)} \left(\frac{PA}{W} \right) \quad (3)$$

where

I_p' = polar moment of inertial of the parts forward of the section
being considered

n = rifling twist in calibers per turn

D_o = outside shell diameter

D_i = inside shell diameter

P = breech pressure

A = gun bore cross-sectional area, and

W = projectile weight

Equation (3) is applicable in cases where the total force is transmitted to the shell wall. The projectile employs a slip obturator which decouples the spin such that the actual angular velocity of the projectile, ω_p , is much less than that of the rotating band. As a result, this tangential shear force is reduced proportionately. The actual tangential force is related to the non-slip force by the ratio of the projectile angular velocity to the band angular velocity. See Reference (1).

$$F_T(\text{slip}) = \frac{\omega_p}{\omega_b} F_T \quad (4)$$

Equation (4) assumes an average or constant angular acceleration.

The nature of the loading on the projectile as a result of the breechblock gas pressure is quite complex and requires careful consideration. Use of the simple relation

$$F = ma$$

in no way describes the true situation. Upon ignition of the gun propellant, shock waves are produced which traverse the explosive chamber and interact with the base of the projectile resulting in shock waves propagating longitudinally through the projectile material. Interactions of repeated shock waves from the explosive and reflected waves within the projectile body produce an unpredictable series of weak and strong shocks (see Figure 2). Of particular importance to this analysis is the overall resultant or transient reverse loading of the material at any one section due to these reflected waves. Joints will be especially affected, particularly if allowed to move, by a slapping of the joint sections. This phenomena has been measured in experiments conducted at NWL. However, at present no method has been formulated to calculate the actual effect the resultant forces have on the material.

Data from these tests indicate that localized accelerations vary greatly over a given time span (5 msec) and generally range as high as 40,000 g in *either* the positive or negative direction (negative indicating rebound). See References (2) and (3). Such a figure would not be a good design criteria, however. Most materials, unless extremely brittle in nature, would not fail under these loads because of the extremely short time span between each loading cycle (50 μ sec). The strength of most metals increases significantly as the strain rate increases. Unless the relationship between yield strength and strain rate are known, an "average" or quasi-static value must be obtained. From the small amount of data available, quasi-static loads for the 8-in. projectiles appear to be approximately 7,000 - 10,000 g, while "rebound" ranges from 1,500 - 2,500 g or 20 - 25 percent of the peak setback forces. Thus if this information is taken to be correct, then the in-barrel rebound force will simply be

$$F_R = .25W'a \quad (5)$$

Normal rebound which occurs at the muzzle may also reach this value, although it depends upon the distribution of the weight in either case. Thus both types of "rebound" were considered concurrently in all analyses and designated with the subscript "R".

A tensile stress in the wall of the shell will result from the rotation of the projectile. Actual determination of this stress would require the use of the thick-wall cylinder equations. This "hoop" stress is very small compared to the stress due to setback, therefore it is possible to use the thin-walled cylinder approximation

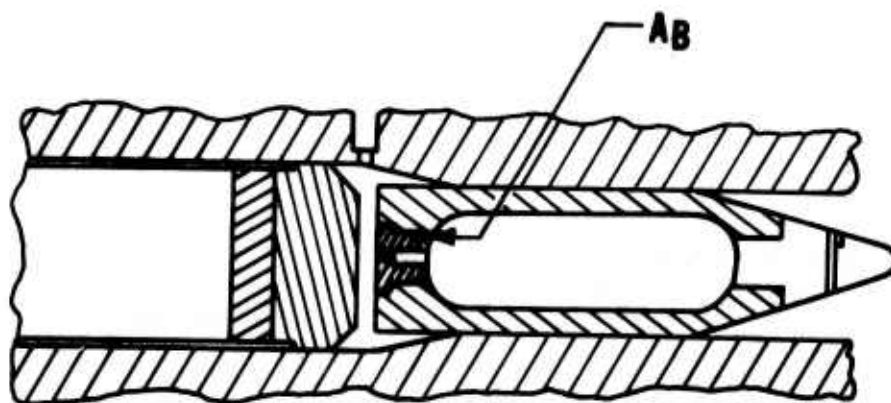
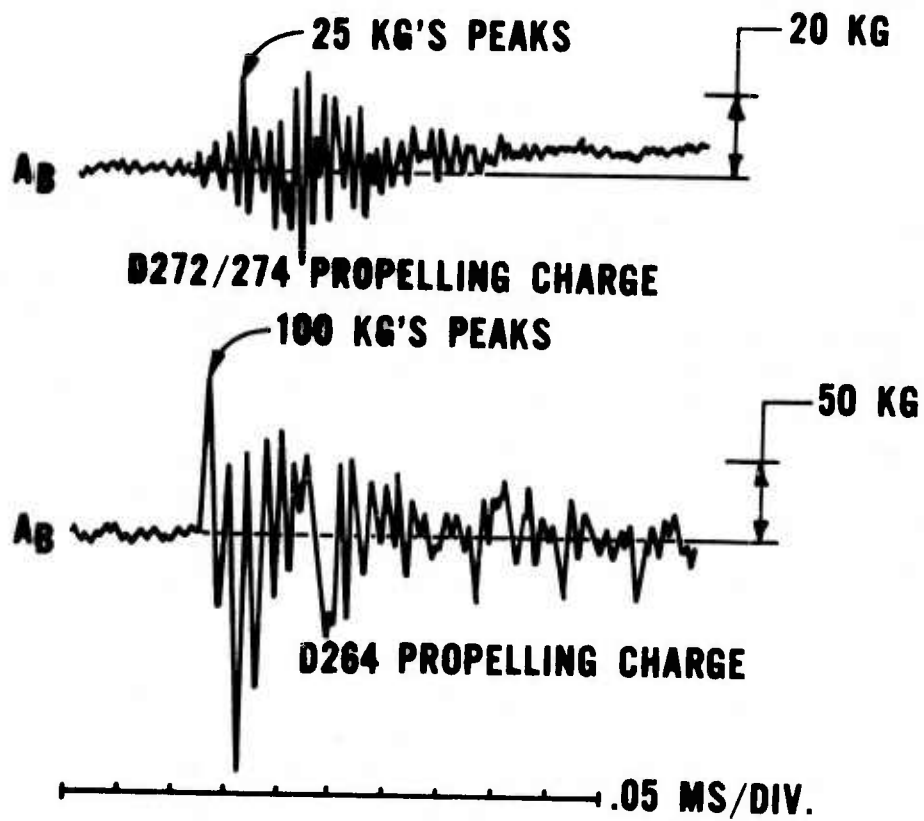


FIGURE 2
Experimental Shock Pressure Pulses

without introducing any significant error. By selecting an elemental mass of the cylinder along its axis and summing forces, it is found that the stress in the wall is

$$S_{2w} = \frac{F_w}{Lt_i} = \frac{12\rho}{g} \left(\frac{R_o + R_i}{24} \right)^2 \omega_p^2 \quad (6)$$

Expressing R_o and R_i in inches and substituting the expression for the angular velocity

$$\omega_p = \frac{24\pi V}{nD} \quad ,$$

then

$$S_{2w} = \frac{12\pi^2\rho}{D^2g} (R_o + R_i)^2 \left(\frac{V}{n} \right)^2 \quad (7)$$

Misalignment of the projectile center of gravity with the gun geometric center will likely cause balloting of the projectile, as a result of the accelerating force creating a moment about the projectile center of gravity. Efforts are in progress to determine the sideload corresponding to this moment experimentally, but results obtained thus far have been inconclusive. The derivation which follows contains some rather broad assumption but its use proved very useful and appeared to provide a good estimate.

If the projectile is represented by a beam simply supported at two points corresponding to the bourrelets, the equation of motion is

$$m\ddot{x} + \zeta\dot{x} + kx = F_o \sin \Omega t$$

where

m = mass,
 ζ = damping coefficient, and
 k = stiffness constant.

One solution of this equation for x , the deflection, is

$$x = \frac{F_o}{k\alpha},$$

where

$$\alpha = \left[\left(1 - \frac{\Omega^2}{\Omega_n^2} \right)^2 + \left(2\zeta \frac{\Omega}{\Omega_n} \right)^2 \right]^{1/2}. \quad (8)$$

For a simply supported beam

$$k = \frac{48EI}{l^3},$$

and for a hollow circular beam the cross-sectional area moment of inertia, I , is

$$I = \frac{\pi}{64} (D_o^4 - D_i^4).$$

It was found that equation (8) could be simplified for gun firing conditions. The vibrating frequency, Ω , is much less than the natural frequency, Ω_n , as witnessed by experimental data. The ratio Ω/Ω_n , then becomes very small such that $\alpha \approx 1$. Solving for F_o , the force then is

$$F_o = xk.$$

Applying this analysis to the 8-Inch Guided Projectile and assuming that the properties and wall dimensions of the warhead apply, then it is found that

$$I = 111 \text{ in.}^4,$$

and

$$k = \frac{48(30 \times 10^6)(111)}{16^3} = 3.91 \times 10^7 \text{ lb/in.}$$

Theoretical calculations indicate also that the maximum deflection of the beam with such a k value would be on the order of

$$x = .001 \text{ in.}$$

Thus for a total projectile weight of 250 lb,

$$F_o = 39,100 \text{ lb.}$$

Application of the force, F_o , at various points on the projectile is accomplished by relating F_o to the moment distribution. Use of the shear-moment diagrams for a 1.0 g loading show that the moment at the forward warhead/fuze joint and the warhead/afterbody joint is, respectively,

$$M_1 = 536 \text{ lb-in.}$$

and

$$M_2 = 506 \text{ lb-in.}$$

The sideload acceleration is

$$a_s = \frac{F_o g}{W} = 156 \text{ g.}$$

thus the total moments

$$M_{x_1} = M_1 a_s = 83,616 \text{ lb-in.}$$

and

$$M_{x_2} = M_2 a_s = 78,936 \text{ lb-in.}$$

The moment is directly related to the stress at any point by the relation

$$S_{11} = \frac{M_x c}{I} \quad (9)$$

where c is the average effective moment arm. The parameters, c and I , are determined readily from the geometry of the section involved, thus the sideload stress can be determined for any section along the axis of the projectile.

In general, the simple stress due to the distribution of a particular force on a relevant area is

$$S = F/A \quad (10)$$

When stresses act on the same planer element of material from orthogonal directions, the principle stresses or maximum and minimum stresses are

$$S_{max}, S_{min} = \frac{S_x + S_y}{2} \pm \left[\left(\frac{S_x - S_y}{2} \right)^2 + S_{xy}^2 \right]^{1/2} \quad (11)$$

where

S_x, S_y are the two dimensional plane stresses, and
 S_{xy} is the 2-D shear stress.

In such cases, the Von Mises Maximum Energy Theory was used to determine if failure (or yielding) would occur. For a three dimensional element, this maximum stress is

$$S^2 = S_1^2 + S_2^2 + S_3^2 - (S_1 S_2 + S_2 S_3 + S_1 S_3) \quad (12)$$

where

S_i = three dimensional principle stresses

and yielding would occur when S equals or exceeds the yield strength of the material.

Of special interest in the analysis were the stresses resulting from thermal effects and the stresses involving pressures on thick-walled cylinders. When a radial internal or external pressure is applied to a thick-walled cylinder, two stresses are produced, a non-uniform tangential or hoop stress, S_2 , and a radial stress, S_3 . If an internal pressure, P_i and an external pressure, P_o are applied to a cylinder with an inner radius and outer radius, R_i and R_o , respectively, the stresses at any point, r , are from the Lamé' equations

$$S_2 = S_{2_t} = [(R_i^2 P_i - R_o^2 P_o) + (P_i - P_o) R_i^2 R_o^2 / r^2] / (R_o^2 - R_i^2) \quad (13)$$

and

$$S_3 = [(R_i^2 P_i - R_o^2 P_o) - (P_i - P_o) R_i^2 R_o^2 / r^2] / (R_o^2 - R_i^2) \quad (14)$$

For the special case when the external pressure, $P_o = 0$, equations (13) and (14) reduce to

$$S_2 = S_{2_t} = R_i^2 P_i (1 + R_o^2 / r^2) / (R_o^2 - R_i^2) \quad (15)$$

and

$$S_3 = R_i^2 P_i (1 - R_o^2 / r^2) / (R_o^2 - R_i^2) \quad (16)$$

These equations indicate that S_{2_t} and S_3 are maximum at the inner surface. Thus for $r = R_i$,

$$S_2 = S_{2_t} = P_i (R_o^2 + R_i^2) / (R_o^2 - R_i^2) \quad (17)$$

and

$$S_3 = -P_i \quad (18)$$

The Lamé' equations are also applicable for press-fit cylinders. If two cylinders with unstressed radii R_{o1} and R_{i1} for the outer cylinder, and R_{o2} and R_{i2} for the inner cylinder are pressed together with a radial interference

$$\delta = R_{o2} - R_{i1}$$

a pressure will result between the two parts of

$$P = \delta E_1 E_2 / [R_c (\gamma E_2 + \beta E_1)] \quad (19)$$

where

$$\begin{aligned} \gamma &= (R_{o1}^2 + R_{i1}^2) / (R_{o1}^2 - R_{i1}^2) + \mu_1 \\ \beta &= (R_{i1}^2 + R_{o2}^2) / (R_{i1}^2 - R_{o2}^2) - \mu_2 \end{aligned}$$

and

$$R_c = R_{o2} - \delta [\sigma_1 / (\sigma_1 + \sigma_2)]$$

and is the radius to the contact surface between the two cylinders (see Appendix D).

The thermal stresses due to restrained expansion of a body (i.e., the normal expansion which would normally occur is prevented) is

$$S_{1t} = E \alpha (T_1 - T_2) \quad (20)$$

where

T_i = temperatures at times τ_1 and τ_2 .

During the analysis, several conditions were assumed:

1. The rotating or obturation band is located on the base of the warhead.

2. The warhead shell is fabricated of steel while the afterbody and G&C housing are aluminum.

3. Friction between the forward bourrelet and the gun bore is negligible.

4. The maximum linear velocity (MCLWT gun) = 2,800 ft/sec.

5. The maximum linear acceleration = 8,000 g.

6. The angular acceleration (no slippage) = 98,000 rad/sec².

7. The maximum angular velocity = 25 rev/sec.

8. The maximum temperature variation is -65° to 160°F.

III. RESULTS AND CONCLUSIONS

Examination of the interface designs indicated that, based on the given conditions and assumptions, two of the designs were adequate while the remainder were either completely incapable of handling the loads encountered or in need of minor design alterations to strengthen areas considered weak. The designs found to be adequate were the two-section inverted Marman band and the four-bolt designs for the afterbody/warhead interface.

Analysis of the two-section inverted band showed that the two .5-in. band connecting bolts would be the first items to experience failure under maximum design load conditions. During gunfiring, the tensile force in the bolts, as a result of rebound and vibration forces acting on the connected projectile components, would be 5,260 lb. Thermal effects due to cooling, however, would add another 18,200 lb tension due to the uneven contraction of the steel and aluminum mating parts. The resultant of these loads is sufficiently below the maximum load capability of the bolts, but since the pretension in the bolts also adds to the total load, it was found that the initial tension applied to the bolts must be limited to 7,000 lb or approximately 700 in.-lb torque. Structurally, all other aspects of the design appear quite adequate with no safety factor being less than 2.0. Details of the analysis are shown in Appendix C. The .75-in. bolts in the four-bolt design would experience a high relative stress of 129,000 psi during rebound. The strength of the bolts chosen, however, is a minimum of 180,000 psi such that a comfortable margin of safety of 39 percent exists. Most other aspects of the design exhibit even greater margins. The reduction in strength of the afterbody due to the removal of material to allow access to the bolts results in a marginal condition with a safety factor of only 1.08. Dynamic load conditions could reduce this margin further, thus the removal of material in this area must be kept to a minimum.

In the analysis of the three-section ring joint for the afterbody/warhead interface, several areas appeared to be inadequate. The shear stress in the rings in the reduced area under the bolt heads could reach 53,650 psi as a result of rebound and other loads thus exceeding the ultimate shear strength of the aluminum material. The force, F_b , in the bolts creating that stress, however, was 23,175 lb which far exceeded the bolt strength of 13,800 lb. Comparison of these two conditions showed that the bolts should experience tensile failure before the shearing stress under the bolt heads could cause failure in the aluminum. High stresses would also occur in the afterbody in the section cut away for the band. The presence of the 3-in.-diameter weight reduction hole resulted in a compressive stress of 86,200 psi, a level which would be beyond the yield point of the material. Adoption of this joint design would necessitate the removal of the cavity in order to reduce the stress to a satisfactory level. It is felt that the other problems

mentioned above could also be overcome. Reduction of the force, F_b , would be accomplished if the torque applied to the bolts could be reduced. Such a reduction, however, would lead to other difficulties, particularly with respect to the sealing of the area under the band from the gun gases. Without an initial tensile force on each bolt of a minimum of 11,000 lb, the band will become loose during setback allowing the gases to apply pressure on the total inside area of the band resulting in sure failure. It would, therefore, be advisable to increase either the size or strength of the bolt and the strength of the band to overcome these deficiencies.

Successful use of press-fitted joints in projectile applications led to the study of the concept for the joints on the 8-in. projectile. However, the results of the analysis indicated two major problems. One problem with the design as presented, and indeed would be present in virtually all such designs, is the high stress concentrations occurring in the outer cylinder. It was estimated that the compressive stresses due to the acceleration and stress concentrations would be 163,000 psi in the outer steel collar. The initiation of cracks in these areas would be highly probable which would lead to failure if the loads are reversed as in rebound. Increasing the thickness of the section, increasing the strength of the material, or incorporating generous fillets at critical points are ways of overcoming this problem. Caution must be used, however, in employing the first two methods since they both have inherent offsetting factors which may lead to even higher stresses. A second problem was indicated in that the radial interference, δ , was not adequate to develop sufficient pressure between the two cylinders to prevent slippage during rebound. The crux of the problem lies in the variations in the two materials involved, particularly with respect to their strengths. It is concluded from the analysis that this problem cannot be overcome under the existing requirements, primarily because of the aluminum and steel properties. Thermal effects due to temperature variations, although always unfavorable, were very nearly negligible. Both of the aforementioned problems occurred in both the afterbody/warhead and warhead/G&C interfaces.

Finally, the analysis of the three-section ring for the warhead/G&C interface revealed that the design is quite adequate from a structural standpoint. Stresses in the G&C aluminum housing reach excessive levels of around 75,000 psi during setback, but a small increase in the thickness of the housing in that area would eliminate problems there. Also, the force in the ring connecting bolts may reach 11,600 lb, thus necessitating a restriction on the initial torque to 170 in-lb.

Figures 3 and 4 summarize the conclusions of the analyses. Plotted on these figures is the ratio of the stress to the local material strength (otherwise called the safety factor) for the weakest area found in each design versus the axial acceleration. Values of the safety factor less than 1.0 indicate yielding of the

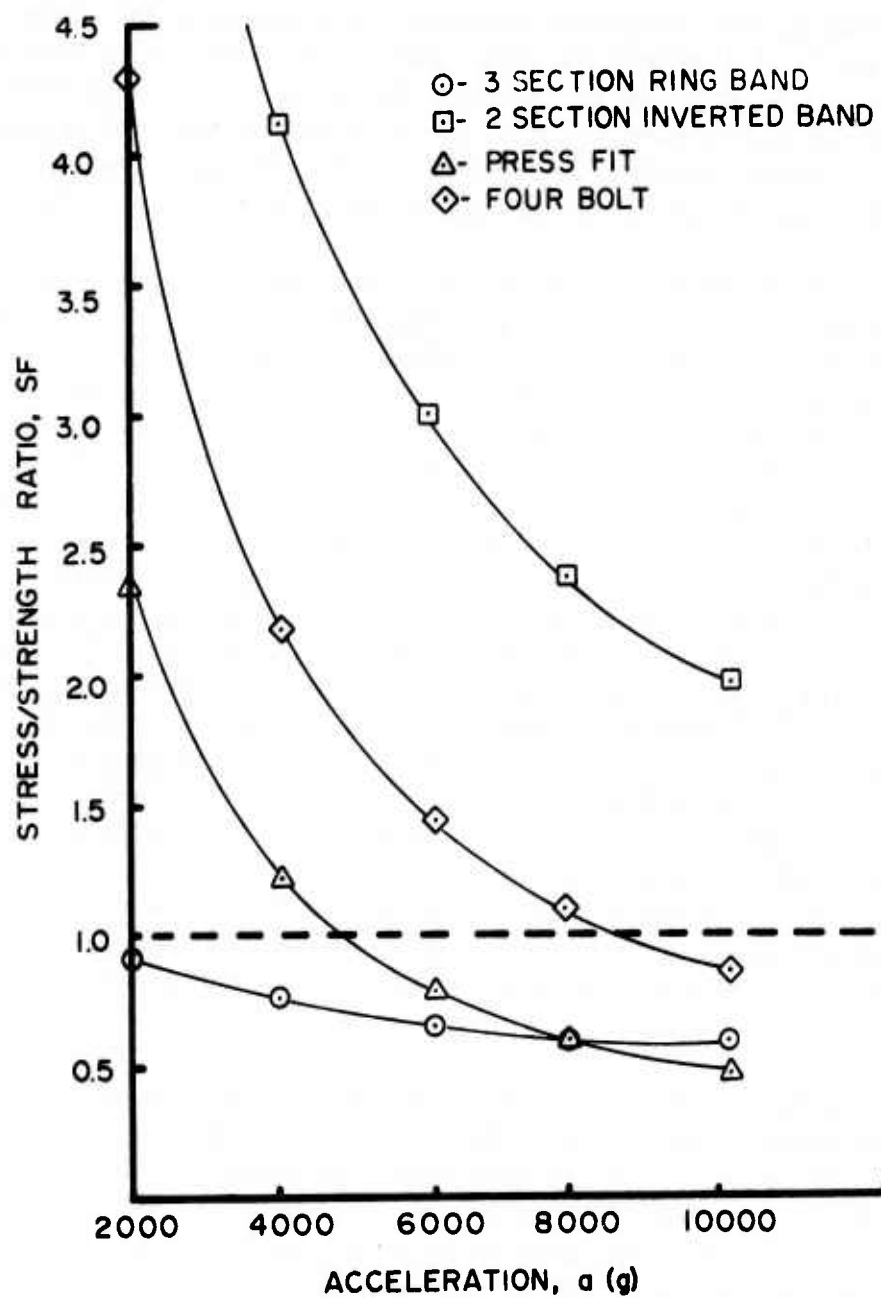


FIGURE 3
Strength Comparison of the
Afterbody/Warhead Interface Joint Designs

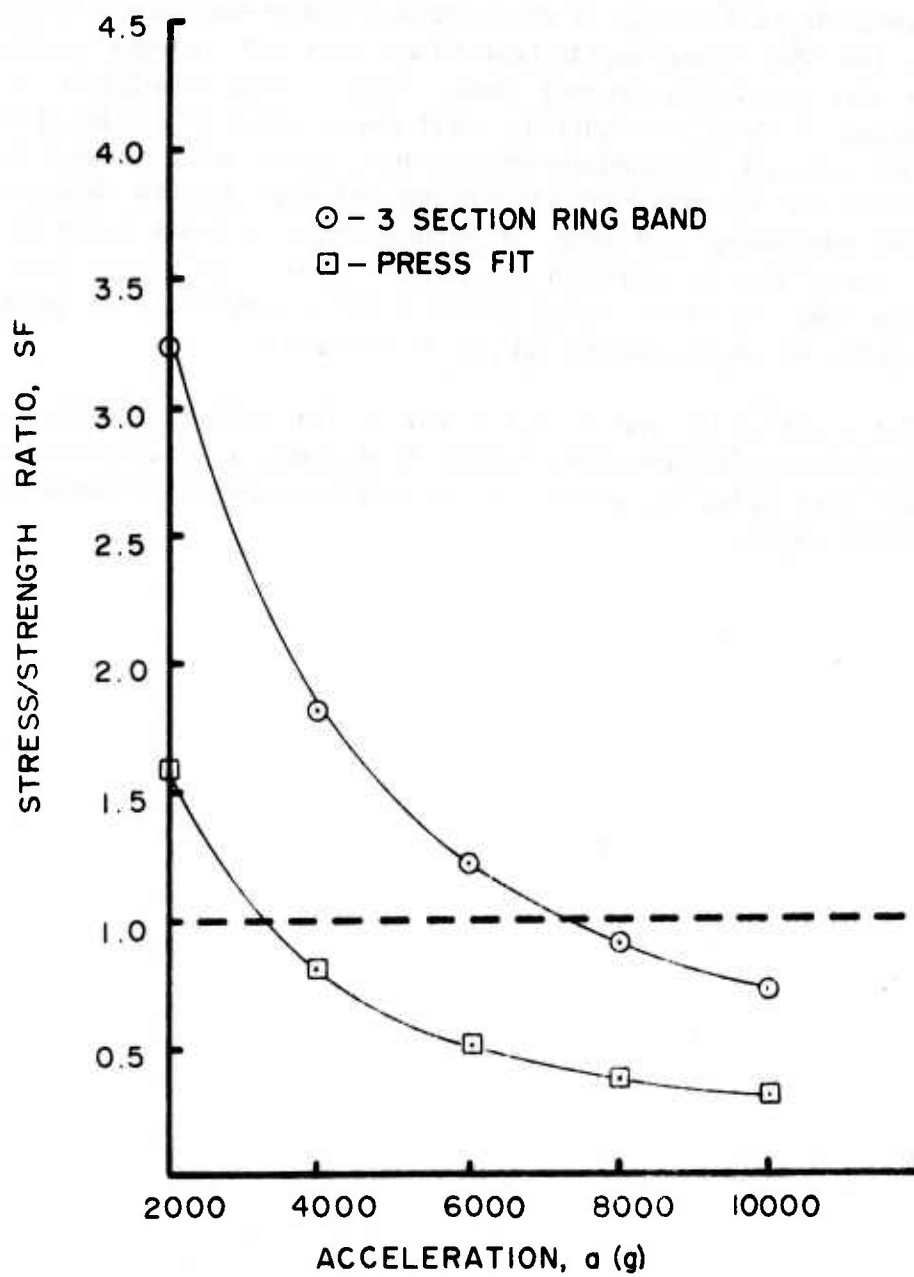


FIGURE 4

Strength Comparison of the
Warhead/Guidance and Control Interface Joint Designs

material which is considered synonymous to failure. The curves were generated by expressing the stress as a function of the acceleration and comparing the resultant stress to the appropriate yield strength of the material, including stress concentration factors, if any. For the comparison, thermal effects were not included in the equations. Thus the two-section inverted band, which is very susceptible to temperature variations, is shown far above the other designs when in actuality it is comparable to the four-bolt configuration when thermal effects are considered. It should also be noted that the probability of correcting the weak areas has obviously been omitted. The three-section ring band, for instance, which is shown having the lowest strength curve, can be improved considerably with only a few minor improvements such that the safety factor, SF, at 6,000 g could easily be made greater than 1.0 while the press-fit design may not be correctable.

Based on the results of the analysis, and in view of the relatively high amount of machining required for the two-section inverted band design, it is recommended that the four-bolt joint design be adapted for the afterbody/warhead interface on the 8-Inch Guided Projectile.

REFERENCES

1. AMCP 706-247, **Engineering Design Handbook**, Ammunition Series, Section 4, *Design for Projection*, pp 177-189.
2. Devost, V. F., **Artillery Projectile Shock**, AD 810 466L, (NOL TR-78-3).
3. Culbertson, D. W. et al, *Investigation of 5-Inch Gun In-Bore Ammunition Malfunctions*, NWL TR-2624, Dec 1971.
4. Timoshenko, S., **Strength of Materials**, Part II, *Advanced Theory and Problems*, Van Nostrand Reinhold Co., New York, 1958. Articles 40-2, 55-7, 82, 87.
5. Shigley, J. E., **Mechanical Engineering Design**, McGraw-Hill, Inc., 1963, pp 170.
6. Baumeister, T. and Lionel S. Marks, **Standard Handbook for Mechanical Engineers**, McGraw-Hill, Inc., 1967.
7. NBS Handbook H28, *Screw-Thread Standards for Federal Services*, Part I, Appendix A-5, U. S. Government Printing Office, Washington, D. C., 1969.

APPENDIX A

**STRUCTURAL ANALYSIS OF THE
BOLTED THREE-SECTION RING – AFTERBODY/WARHEAD INTERFACE DESIGN**

STRUCTURAL ANALYSIS OF THE BOLTED THREE-SECTION RING – AFTERBODY/WARHEAD INTERFACE DESIGN

The geometry of the design analyzed is shown in Figure A-1. For this location, the weight of the metal parts forward of the section is approximately

$$W' = 195 \text{ lb}$$

(afterbody weight, $W'' = 55 \text{ lb}$).

The total weight of the aluminum joining band is

$$W_B = 4.4 \text{ lb}.$$

The possible modes of failure appear to be (refer to Figure A-1):

1. Failure of the joining band at its center in tension;
2. Failure of the band in shear at the bolt seating surface such that the bolt heads pull through;
3. Failure of the bolts in tension;
4. Failure of the connecting rim on the afterbody;
5. Failure of the afterbody in compression due to the presence of the internal hole;
6. Yielding at bearing surface;
7. Failure of the joining rings in tension.

In the analysis of this design, it should be noted that the gaps before and aft of the rings will expose much of the rings to the breech chamber gas pressure. It is assumed that this will be eliminated and thus is not considered in the equations for stress.

Looking first at the lateral cross-section of the joining ring (at its center), the following stresses were determined.

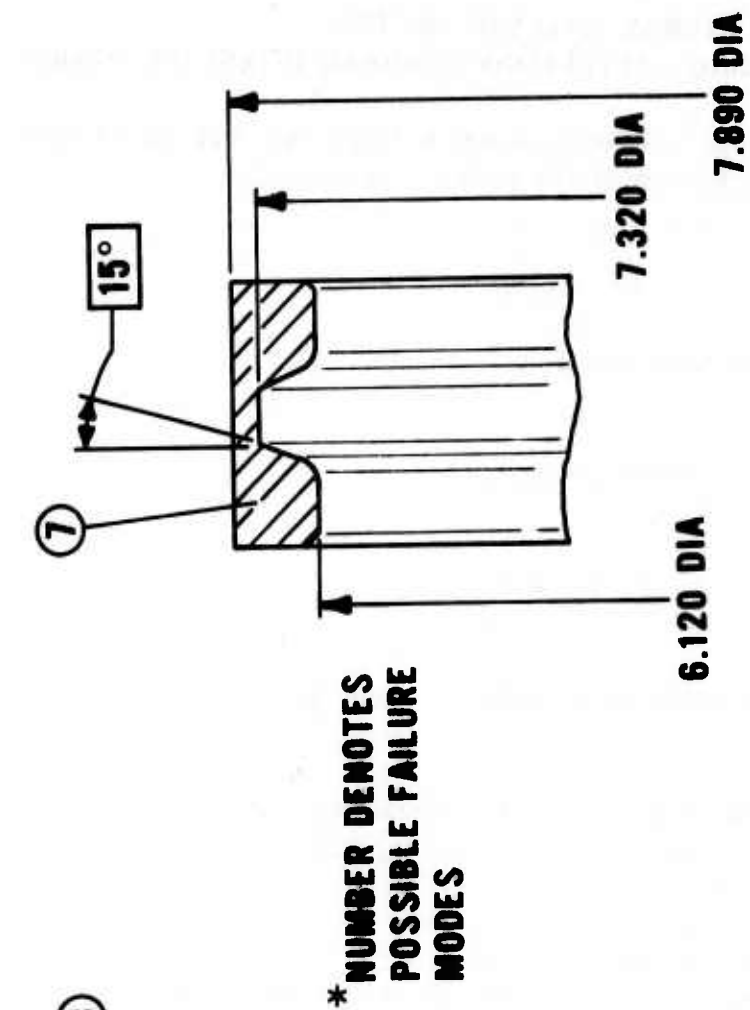
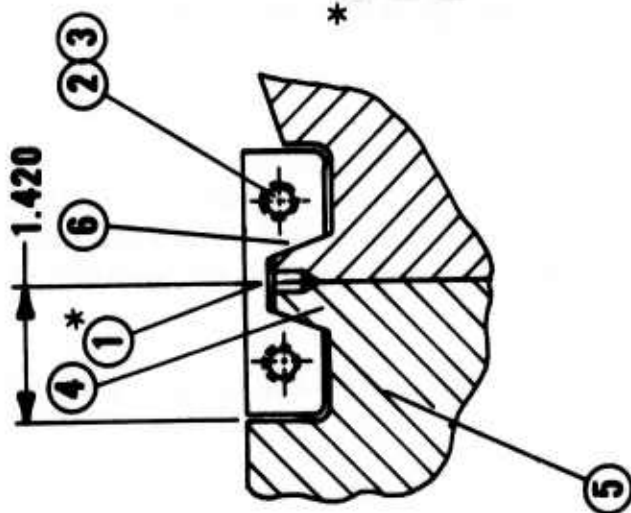


FIGURE A-1
Three-Section Ring Joint for the
Afterbody/Warhead Interface



The tensile stress in the band due to setback of the rings, from equations (1)* and (10):

$$F_{s_b} = 1/2 (4.4)(8,000)$$

$$= 17,600 \text{ lb}$$

$$A' = \frac{\pi}{4} (7.89^2 - 7.32^2) = 6.81 \text{ in.}^2$$

$$S_{1_b} = 2,584 \text{ psi}$$

The hoop tension in the band due to angular rotation of the projectile is from equation (6):

$$S_{2_w} = 317 \text{ psi}$$

If the six bolts in the joining band are torqued to 80 percent of the ultimate tensile strength, which is the recommended torque for maximum fatigue life of the bolt, a tensile force in the band will result:

$$F = 2(.8)(13,800) = 22,080 \text{ lb}$$

The stress resulting in the band due to the force, F, for a cross-sectional area of

$$A_b = 1.98 \text{ in.}^2$$

is

$$S_{2_b} = 11,152 \text{ psi.}$$

*Refer to main text of report for equations.

A hoop shearing stress will be present in the rings due to the tangential force imparted by the rotating band. Assuming no slippage between mating surfaces, the applicable area over which the force is distributed is

$$A = \frac{\pi}{4}(7.89^2 - 3.0^2) = 41.8 \text{ in.}^2$$

if a 3-in. interior hole is present in the afterbody. Assuming that the polar inertia of the metal parts forward of this section is

$$I'_p = 1,900 \text{ lb-in}^2$$

and the calibers per revolution of the gun bore are

$$n = 25 ,$$

then from equations (3), (4), and (10), the stress is

$$S = 760 \text{ psi} .$$

During "rebound", a hoop tensile stress will be observed in the rings. This stress results from the radial component of the rebound and moment forces acting on the 15 degree slanted surfaces. From equations (1) and (5), the rebound force is

$$F_R = .25W''a = 110,000 \text{ lb} .$$

To obtain the force due to the sideload bending moment it is simply necessary to find the stress and apply it to the appropriate area. Considering the tensile moment only, the force will be transmitted totally through the joining band. For the band the moment of inertia is

$$I = 49.3 \text{ in.}^4 ,$$

which indicates that the stress is [from equation (9)]

$$S_{11} = 6,084 \text{ psi.}$$

The stress is being applied on only half of the band at any given instant, while the other half is in compression, thus the area over which the stress exists is

$$A = 1/2A' = 3.405 \text{ in.}^2 ,$$

resulting in a total bending moment force of

$$F_m = 20,700 \text{ lb.}$$

The contact area of the rings with the body at the slanted surfaces, assuming both surfaces to be perfectly flat, is approximately

$$\begin{aligned} A &= \pi(R_h^2 - R_l^2)/\cos \theta \\ &= 11.0 \text{ in.}^2 \end{aligned}$$

where

$R_h = 3.635$ = radius to highest contact point, and

$R_l = 3.135$ = radius to lowest contact point.

Thus, the pressure on this surface due to the rebound and bending moment is

$$P_{avg} = 11,880 \text{ psi.}$$

The radial component of this pressure is from the geometry of the design

$$\begin{aligned} P_i &= P \cos \theta \sin \theta, \quad \theta \neq 0^\circ, 90^\circ \\ &= 2,970 \text{ psi.} \end{aligned}$$

From equation (16), the hoop stress resulting from this pressure is

$$S_t = 11,940 \text{ psi.}$$

Also during rebound, a tensile stress in the axial direction will exist due to the rebound force

$$S_{1b} = 16,150 \text{ psi.}$$

The total stresses in the rings then are during rebound:

$$\begin{aligned} S_1 &= S_{1b} + S_{11} \\ &= 22,230 \text{ psi (longitudinal)} \end{aligned}$$

$$\begin{aligned} S_2 &= S_{2b} + S_{2w} + S_t \\ &= 23,409 \text{ psi (hoop)} \end{aligned}$$

$$S_s = S = 760 \text{ psi (shear)}$$

and during setback;

$$S_1 = S_{1_b} + S_{1_1}$$

$$= 8,668 \text{ psi}$$

$$S_2 = S_{2_b} + S_{2_w}$$

$$= 11,469 \text{ psi}$$

$$S_s = S = 760 \text{ psi}$$

Neglecting the shear as small, and setting S_1 and S_2 equal to the principle stresses, the maximum energy stress is from equation (12), for rebound:

$$S = 22,842 \text{ psi}$$

resulting in a safety factor of

$$SF = 3.15$$

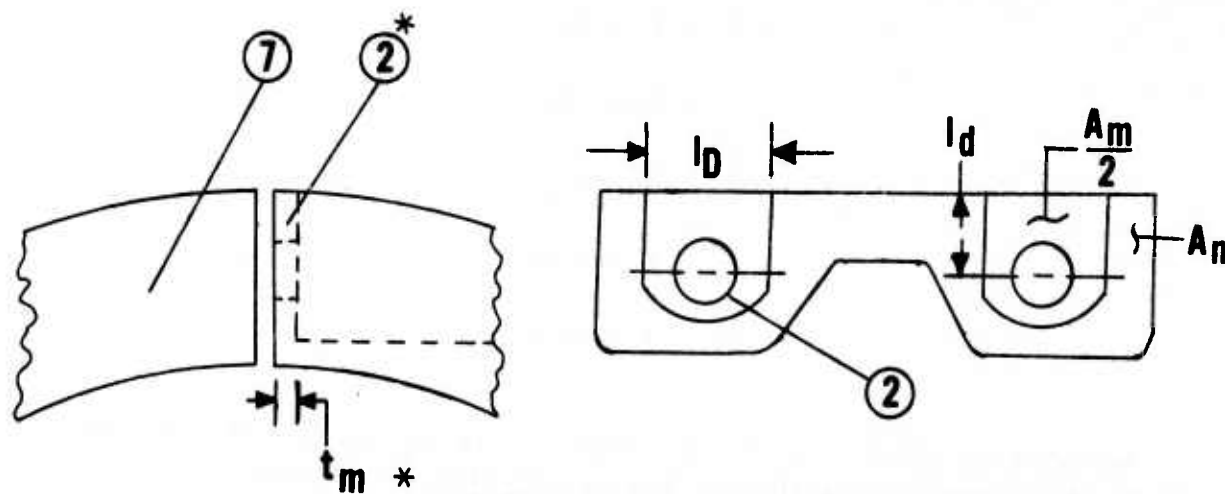
and for setback:

$$S = 10,357 \text{ psi}$$

and

$$SF = 6.95.$$

Figure A-2 is a sketch of the area under consideration regarding failure modes (2) and (7).



NUMBER DENOTES POSSIBLE FAILURE MODES

FIGURE A-2

Cross Section of the Band at the Bolt Stand

A_m is the area removed to allow the seating of the bolts and t_m is the thickness of the bearing area:

$$\begin{aligned} t_m &= .250 \text{ in.} \\ L_d &= .475 \text{ in.} \\ L_D &= .675 \text{ in.} \end{aligned}$$

Thus the total area removed for the bolts is

$$\begin{aligned} A_m &= 2 \left[L_d L_D + .5\pi \left(\frac{L_D}{2} \right)^2 \right] \\ &= 1.00 \text{ in.}^2. \end{aligned}$$

The remaining cross-sectional area of the band is

$$A_n = A_b - A_m = .98 \text{ in.}^2.$$

During rebound, the hoop tension in the band will be

$$\begin{aligned} F &= S_2 A_b \\ &= 46,350 \text{ lb.} \end{aligned}$$

At this location the area is reduced, thus the stress is

$$S_{2_n} = 47,300 \text{ psi.}$$

The force, F , will be distributed to the two connecting bolts and over the bolt head bearing area such that the actual force in the bolts will be

$$F_b = .5F = 23,175 \text{ lb.}$$

The diameter of the bolt head is .55 in., thus the band-shear area under the bolt head which will resist F_b is

$$A_t = \pi d t_m = .432 \text{ in.}^2,$$

resulting in a stress in the band around the bolt seating area of

$$S_{2_m} = 53,650 \text{ psi}$$

such that

$$SF = 0.82.$$

Another possible mode of failure would appear to be across the rim of the afterbody shown as area A_s in Figure A-3. The rebound force, F_R , and the moment force, F_m , will result in a forward pulling of the band by the warhead

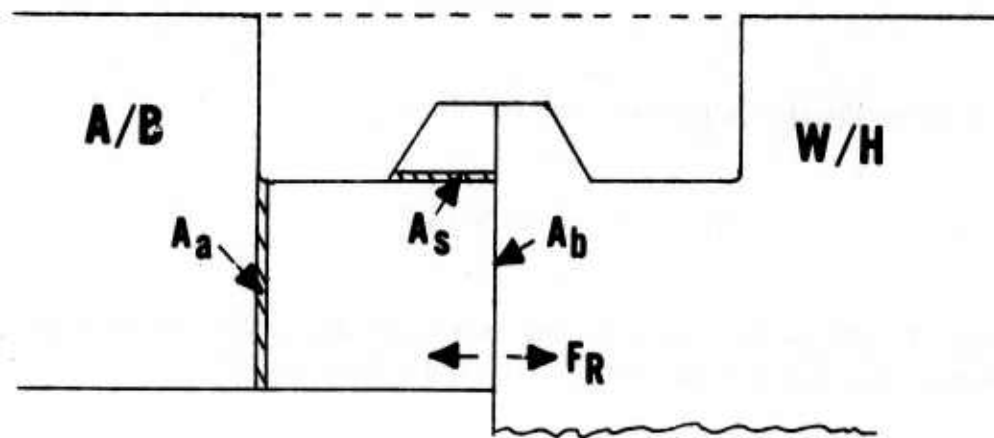


FIGURE A-3

Cross Section of Joined Bodies at Joint

which in turn will produce a shearing force acting on the rearward slanted surface. The shearing stress realized for $A_s = 8.43 \text{ in.}^2$ is

$$S_s = 15,500 \text{ psi.}$$

If an interior cavity is present in the afterbody, the cross-sectional area, A_a , in Figure A-3 is reduced to

$$A_a = 22.16 \text{ in.}^2.$$

The setback force on area, A_a , will result from the chamber pressure due to the location of the obturating band. The force due to the pressure is simply

$$F = PA.$$

However, due to the time dependency of P , the effect of the rotating band must be considered. In this case the force applied to the band will tend to pull the afterbody. Thus

$$F_s = P(A - A_1).$$

Assuming a peak breech pressure of 40,000 psi, and determining the maximum cross-sectional area aft of the interface,

$$A = 49.02 \text{ in.}^2$$

and

$$A_1 = 1.25 \text{ in.}^2,$$

then

$$F_s = 1.91 \times 10^6 \text{ lb.}$$

Therefore, the maximum stress at this section is

$$S_1 = 86,200 \text{ psi}$$

which gives a safety factor of

$$SF = .84.$$

Without the 3-in. interior cavity, the stress is

$$S_1 = 65,350 \text{ psi}$$

and

$$SF = 1.10.$$

The bearing area, again assuming an interior cavity in the afterbody, is greater than A_a due to the increased radius of the rim. The bearing area, A_b , is 34.79 in.² such that the bearing stress is

$$S_1 \text{ (bearing)} = 54,908 \text{ psi}$$

and

$$SF = 2.24.$$

In summary, all of the apparent modes of failure were examined for the three ring band design. The ring itself is sufficiently strong except in the area where material is removed for the seating of the bolts. The calculations indicate that the tensile (hoop) stresses in that area exceed the strength of the material in both the total cross-sectional area of band, A_n , where the stress reaches a maximum of

$$S_{2_n} = 47,300 \text{ psi,}$$

and in the bolt seating area where the shear stress may reach a value of

$$S_{2_m} = 53,850 \text{ psi,}$$

exceeding the ultimate shearing strength of the material which is 44,000 psi. With the weight-reduction hole in the afterbody, the stress was

$$S_1 = -86,200 \text{ psi,}$$

resulting in a safety factor of only

$$SF = .84$$

indicating that the cavity must be reduced.

The only other area of concern was in the band bolts in which the tensile force reached a maximum of

$$F_b = 23,175 \text{ lb}$$

in each bolt. This far exceeds the ultimate strength of the bolts for which

$$F_{max} = 13,800 \text{ lb.}$$

One method of reducing the force, F_b , would be to reduce the initial torque on the bolts. However, even if no initial torque is applied to the bolts, the force, F_b , as a result of rebound would be more than the maximum load the bolts can safely withstand, while the 80 percent torque is necessary to prevent the joint from becoming loose during gunfiring. Furthermore, the stress due to this force (F_b) on the shear area under the bolt head, A_t , would be

$$S_{2m} = 53,650 \text{ psi.}$$

which is 1.31 times the shear strength of the aluminum.

To overcome these deficiencies, it would be necessary to

1. Increase the material strength of the band, and
2. Increase the bolt size or strength.

APPENDIX B

STRUCTURAL ANALYSIS OF THE BOLTED THREE-SECTION RING – WARHEAD/GUIDANCE AND CONTROL INTERFACE DESIGN

STRUCTURAL ANALYSIS OF THE BOLTED THREE-SECTION RING - WARHEAD/GUIDANCE AND CONTROL INTERFACE DESIGN

The geometry of this design as analyzed is shown in Figure B-1. Generally, similar forces are present at this location as were felt by the warhead/afterbody joint. For this location,

and

$$W' = 43.5 \text{ lb}$$

$$I_p' = 380 \text{ lb-in}^2.$$

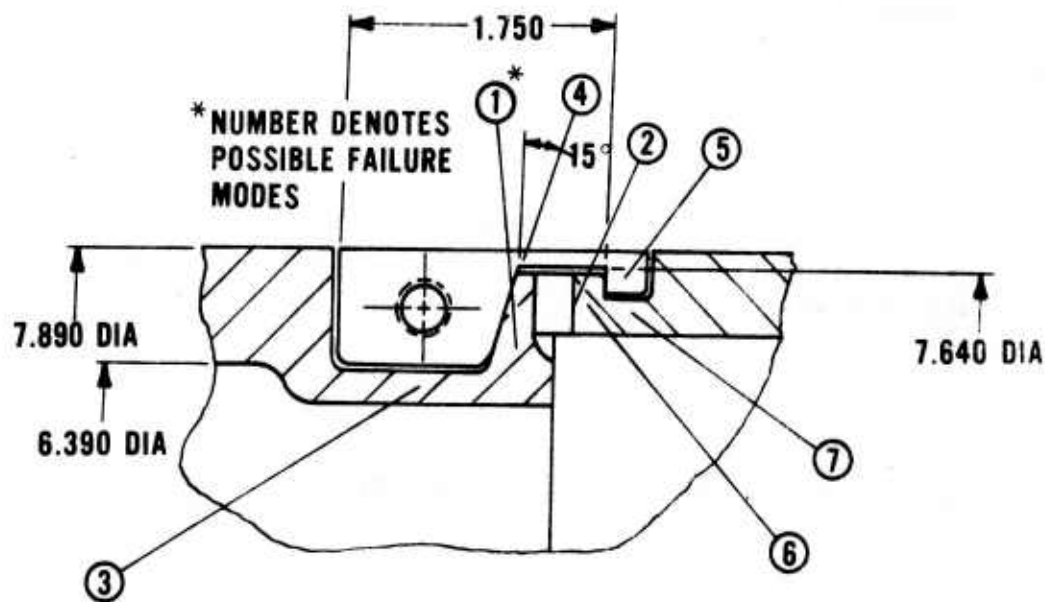


FIGURE B-1

Three-Section Ring - Warhead/G&C Design

The possible modes of failure appear to be, upon looking at Figure B-1:

1. Failure of the warhead rim;
2. Failure of the G&C bearing surface;
3. Failure of the warhead due to setback;
4. Failure of the joining band in tension;

5. Failure in shear of the forward lip of the band;
6. Failure in shear of the G&C housing lip;
7. Failure of G&C housing in compression.

Once again, assuming that the maximum axial acceleration is 8,000 g, the primary forces acting at this section are:

for setback [Equation (1)],*

$$F_S = 348,000 \text{ lb ;}$$

rebound,

$$F_R = 87,000 \text{ lb ;}$$

tangential shear,

$$F_T = 5,800 \text{ lb ;}$$

and hoop tension due to spin,

$$F_W = 4,935 \text{ lb .}$$

The moment force can be calculated from equation (9). For

$$\begin{aligned} I &= \frac{\pi}{64} (7.95^4 - 6.474^4) \\ &= 110 \text{ in.}^4 \end{aligned}$$

and

$$M = 83,616 \text{ lb-in., } C = 3.5 \text{ in.,}$$

* Refer to main text of report for equations.

then

$$S_{11} = 2,660 \text{ psi ,}$$

Since for plane stress

$$F = SA$$

then

$$F_m = 22,240 \text{ lb.}$$

Thermal stresses were considered separate from the stresses due to gunfiring. The areas examined in relation to the aforementioned modes of failure are shown in Figure B-2.

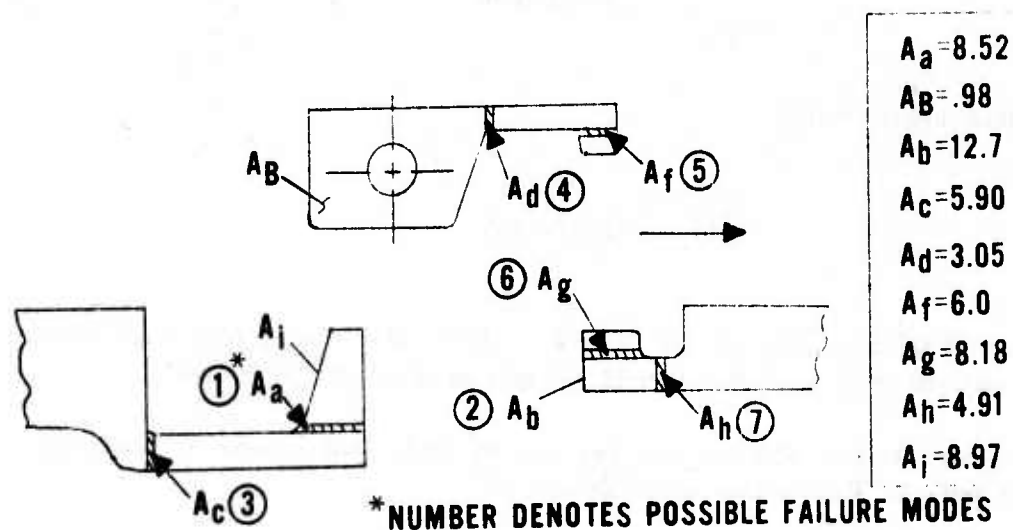


FIGURE B-2

Three-Section Ring Stress Areas

For failure mode (1), the area at the base of the warhead rim, A_a , was examined for static strength. Area A_a was calculated to be

$$A_a = 8.52 \text{ in.}$$

The area appears to be highly stressed during both setback and rebound. During setback, the maximum stress resulting from the setback force and the moment is [from Equation (10)]

$$S_{1s} = 43,450 \text{ psi.}$$

For rebound, the total force if the moment force is tensile,

$$\begin{aligned} F &= F_R + F_m \\ &= 109,340 \text{ lb.} \end{aligned}$$

which indicates that

$$S_{1r} = 12,800 \text{ psi.}$$

The shearing stress, S_{1s} , will not be of any concern since this section is designed for a steel material with a minimum yield strength in shear of 75,600 psi.

The bearing surface between the two joined shells will observe a maximum stress during setback. This surface totals an area of

$$A_b = 12.7 \text{ in.}^2.$$

Considering the maximum stress condition where

$$\begin{aligned} F &= F_s + F_m \\ &= 370,240 \text{ lb.} \end{aligned}$$

the simple stress would be

$$S_{1s} = 29,150 \text{ psi.}$$

Failure mode (3) would most likely occur near area A_C . Reference (4) indicates that stress concentrations are likely in this area near the .10-in. radius located directly above the area. The strength reduction factor for this geometry and material is

$$K_f = 1.5 ,$$

which would result in a maximum peak stress during setback, for $A_C = 5.90 \text{ in.}^2$;

$$\begin{aligned} S_{1s} &= K_f F/A \\ &= 94,100 \text{ psi} \end{aligned}$$

and

$$SF = 1.34 \text{ (steel).}$$

Failure mode (4) could result from three possible sources: (1) setback of the rearward portion of the joining rings, (2) tension resulting from the rings being tightened to draw the projectile units together, and (3) rebound of the projectile units, or a combination of these. The area, A_d , was determined to be 3.05 in.^2 . Setback of the rearward portion of the band is ($W'_B = 4.9 \text{ lb}$),

$$F_s = 39,200$$

$$\text{for which } S_{1s} = 12,900 \text{ psi.}$$

If the .375 connecting bolts are torqued as before to 80 percent of the maximum allowable, the force in each bolt would be

$$F_{tb} = 11,040 \text{ lb} ,$$

which would result in an overall average stress in the ring for a cross-sectional area,
 $A_B = .98 \text{ in.}^2$,

$$S_2 = 11,265 \text{ psi} .$$

The hoop stress, S_2 , can be related by equation (17) to the radial component of the pressure, P_i , being exerted on the surface consisting of area, A_i . Solving for the normal pressure on surface, A_i ,

$$\begin{aligned} P &= P_i \csc \theta \\ &= 9,044 \text{ psi} \end{aligned}$$

or, for $A_i = 8.97 \text{ in.}^2$,

$$F_i = 81,000 \text{ lb}$$

which has the force component in the X, or axial, direction

$$F_1 = 78,200 \text{ lb} .$$

The longitudinal tensile stress in the rings due to the force, F_1 ,

$$S_{1t} = 25,600 \text{ psi} .$$

The longitudinal stress in the rings due to the rebound force, F_R , is

$$S_{1r} = 28,500 \text{ psi}$$

and the maximum tensile stress due to moment is

$$S_{11} = 7,300 \text{ psi}.$$

The resulting peak tensile stress in the band during rebound is then

$$\begin{aligned} S_1 &= S_{1t} + S_{1r} + S_{11} \\ &= 61,400 \text{ psi} \end{aligned}$$

and

$$SF = 1.17.$$

The forward lip of the joining ring was considered for possible failure in shear. The total maximum force acting on the area, A_f , is

$$F = F_R + F_m + F_l = 187,440 \text{ lb}.$$

Thus, if $A_f = 6.00 \text{ in.}^2$

$$S_s = 31,240 \text{ psi}$$

and

$$SF = 1.41.$$

The rearward rim located on the G&C housing is also influenced primarily by the total force, F , above and, for $A_g = 8.18 \text{ in.}^2$,

$$S_s = 22,900 \text{ psi}$$

and

$$SF = 1.92.$$

For failure modes (4), (5), and (6) involving areas, A_d , A_f , and A_g , respectively, the forces occurring during setback were much smaller than those occurring during rebound and thus were not stated.

Finally, the possible failure of the G&C housing due to setback was examined. A cross section was taken at area A_h for consideration. The maximum force that could occur would be

$$\begin{aligned} F &= F_s + F_m \\ &= 370,240 \text{ lb.} \end{aligned}$$

Thus, for $A_h = 4.91 \text{ in.}^2$,

$$S_{1s} = 75,400 \text{ psi.}$$

It is interesting to note at this point that some thermal effects will arise under maximum conditions. Cooling of the area to $T = -65^\circ\text{F}$ will result in a contraction of the rims on the warhead and G&C of only 72 percent of that which would be experienced by the band. A tightening of the band will result, accompanied by increased stresses. The tensile stress in the band rings in the axial direction was approximated using equation (20)

$$S_{1t} = 5,100 \text{ psi}$$

and in the circumferential direction

$$S_2 = 2,088 \text{ psi.}$$

During the heating cycle with the temperature rising to 160°F, the opposite effect is observed resulting in an equivalent reduction in the respective stresses.

Looking at the band cross-section, the head of the .375-in. bolts will require material to be removed from the band to provide a seating surface and a means for installation. The area required for the bolt is

$$A_b = .50 \text{ in.}^2$$

such that the remaining band area is

$$A_m = .48 \text{ in.}^2$$

As a result, an 11,000 lb initial tensile load in the bolts will result in a tensile stress of

$$S_2 = 23,000 \text{ psi}$$

at the band seat. In addition, the radial force on the band due to rebound and vibration will induce additional stresses which from equation (17) is

$$S_{2_r} = 13,800 \text{ psi } (F_{2_r} = 13,500 \text{ lb})$$

in the normal band cross section and

$$S'_{2_r} = 28,200 \text{ psi}$$

at the reduced section. The total of these stresses (S_2 and S'_{2_r}) is

$$S = 51,200 \text{ psi}$$

and

$$SF = 1.41 .$$

Tension in the band will also produce shear stresses around the bolt head.

The head of the bolt will have a bearing circumference of 2.12 in. on the seating surface of the band. The thickness of the band, t , at this point is .25 in. thus producing a shearing area of

$$A_{\text{shear}} = .53 \text{ in.}^2 .$$

The tensile force in the band and the tensile load in the bolt will give a total load on this section

$$F_1 = 22,600 \text{ lb} ,$$

thus giving a total shear stress of

$$S_s = 46,200 \text{ psi} .$$

No problem is likely here, however, in that the maximum strength of the bolts in tension is 13,500 lb. Thus if the tensile force in the bolts does approach F_1 above, the bolts would be the first to experience failure.

Summarizing, it appears that some problem areas may exist with the design of the areas comprised of the aluminum material while the steel warhead design is quite adequate. During setback, a possible problem may arise in the narrowed section of the G&C housing if the balloting of the projectile during its in-bore travel is great. The peak stress in this area could reach

$$S_1 = 75,400 \text{ psi}$$

which is slightly greater than the minimum compressive strength of the material. The only other area of concern is the ring-joining bolts. The tensile force occurring in the band and bolts as a result of the rebound force is

$$F = 11,600 \text{ lb.}$$

The ultimate strength of the bolts is 13,800 lb tensile, thus the initial tension on the bolts must be restricted to 2,300 lb which is approximately 170 lb-in. torque.

APPENDIX C

**STRUCTURAL ANALYSIS OF THE BOLTED
TWO-SECTION INVERTED RING DESIGN**

STRUCTURAL ANALYSIS OF THE BOLTED TWO-SECTION INVERTED RING DESIGN

A two-ring joint design was also proposed for the warhead/afterbody joint which was similar to the three ring Marman design. The joint consisted of interlocking teeth with a steel band around the center such that the band pushed on the teeth rings rather than pulled as in the three-ring concept. Therefore it was called the "inverted Marman band". The design as analyzed is shown in Figure C-1.

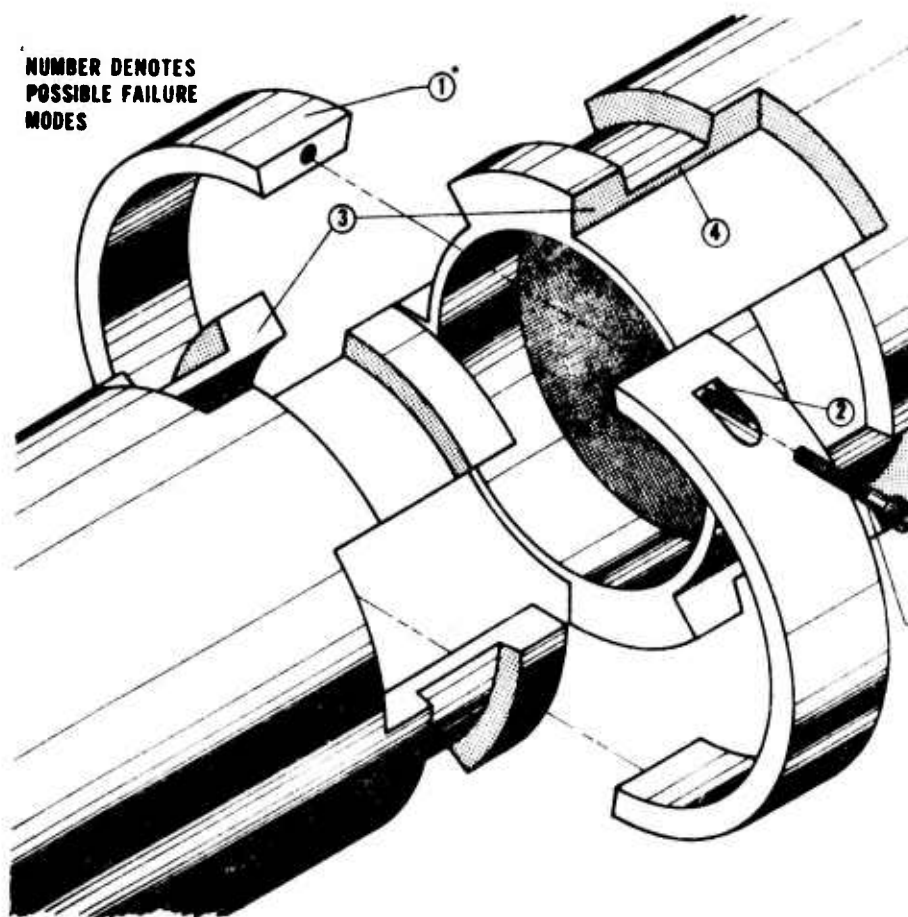


FIGURE C-1

Perspective View Inverted Ring
Band - Warhead/Afterbody Design

Again from equation (1)*, the setback force at the joint force is

$$F_{S_{MAX}} = 1.56 \times 10^6 \text{ lb.}$$

which would include the setback of the band itself. Also, the maximum "quasi-static" rebound force is

$$F_R = .11 \times 10^6 \text{ lb.}$$

The tension in the band due to angular rotation, from equation (6), is equal to

$$F_W = 425 \text{ lb}$$

and the tangential force attempting to spin the remainder of the projectile is [equations (3) and (4)]

$$F_{T_{slip}} = 26,000 \text{ lb.}$$

Finally, the force due to the vibrational moment will be approximately the same as found for the three ring band

$$F_m = 20,700 \text{ lb.}$$

With the forces determined, various cross sections of the joint area were analyzed to determine the effect of the forces as before. The possible modes of failure for this design (Figure C-1) are:

* Refer to main text of report for equations.

1. Failure of the joining band in tension;
2. Failure of the seating surface of the band for the .5-in. bolts;
3. Failure in shear of the warhead or afterbody rims;
4. Failure of the warhead section;
5. Failure of the .5-in. bolts.

The weakest area of the band would be in the area of the connecting bolts as shown in Figure C-2. The head of the required .5-in. bolts will require an area to be taken out of the band to provide a seating surface and means for installation. The loss of this area will greatly decrease the strength of the corresponding section of the band.

The total area of the band cross section is

$$A = .837 \text{ in.}^2.$$

The area required for the bolt head is

$$A_n = .614 \text{ in.}^2.$$

$$\text{Such that } A_m = A - A_n = .217 \text{ in.}^2.$$

The band probably will not be effected by setback or tangential forces. The stress due to the wall tension is also negligible. The .5-in. bolts have an ultimate tensile strength of 25,000 lb and if the bolts are tightened to 10,000 lb initially, the stress in the band on area A_m then will be

$$S_{2b} = \frac{F_B}{A_m} = 46,100 \text{ psi.}$$

In addition, a radial force on the sides of the band due to the rebound and vibration of the afterbody will induce tension in the band. The stress is, for a band slope side area of $A_i = 20.5 \text{ in.}^2$ and a slope of 15° [equation (17)],

$$S_{2r} = 6,285 \text{ psi (at area A)}$$

and

$$S_{2r} = 24,240 \text{ psi (at area } A_m \text{)}.$$

The worse case would occur if these tensile stresses (S_{2b} and S_{2r}) added such that

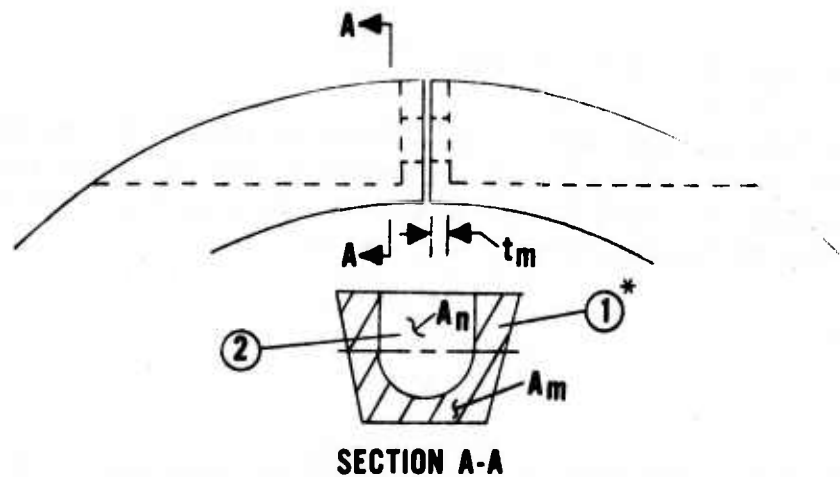
$$S_T = 70,340 \text{ psi}$$

at area A_m , which is comfortably below the yield strength of the 4340 steel band where

$$\sigma_y \text{ min} = 150,000 \text{ psi ,}$$

thus

$$SF = 2.13.$$



* NUMBER DENOTES POSSIBLE FAILURE MODES
FIGURE C-2

Cross Section of Ring Band for
Inverted Design

A check was made to see if the head of the bolt would pull through the bearing surface of the ring as had occurred recently with a previous design.

For $t_m = .25$ in. (see Figure C-2), and a bolt head diameter, $d = .80$ in.

$$A_s = .63 \text{ in.}^2,$$

$$F = S_T A_m = 15,264,$$

$$S_s = 24,230 \text{ psi},$$

and

$$SF = 4.54.$$

Another area to consider is at the base of the rims of both the afterbody and the warhead. This area is labeled A_s in Figure A-3. As before this area is affected most directly by the rebound force, F_R , and the moment force, F_m . The total areas based on the dimensions shown in Figure C-3 are

$$A_{s_{A/B}} = 7.091 \text{ in.}^2$$

and

$$A_{s_{W/H}} = 4.791 \text{ in.}^2$$

Thus resulting in

$$S_{s_{A/B}} = 18,430 \text{ psi},$$

$$SF = 2.39,$$

$$S_{s_{W/H}} = 27,280 \text{ psi},$$

and

$$SF = 4.03.$$

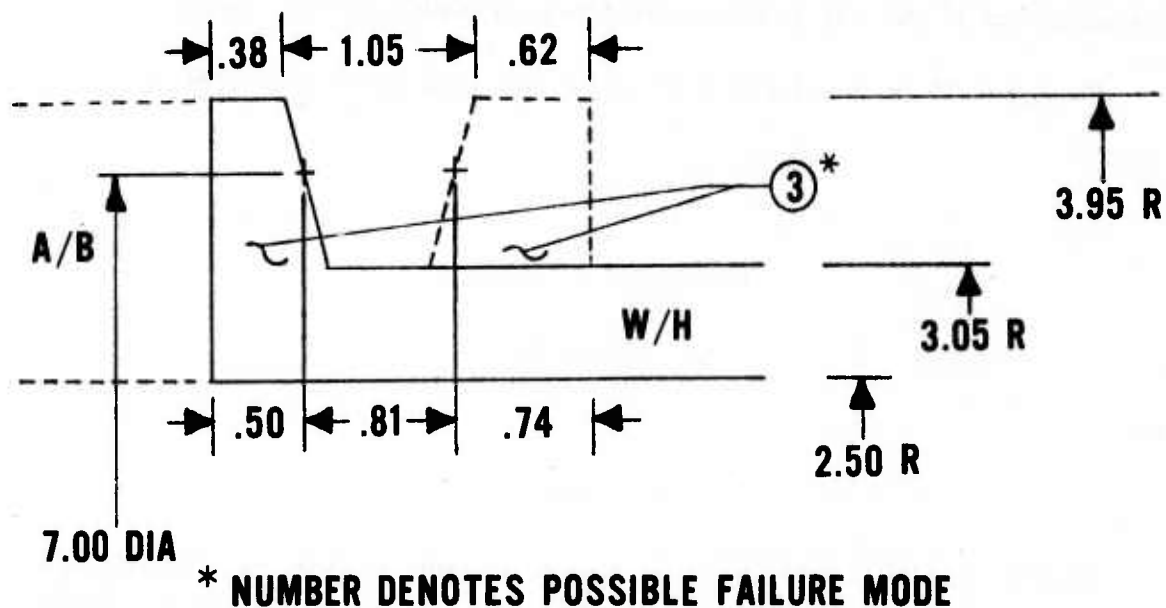


FIGURE C-3

Cross Section of Inverted Design

The steel warhead section will have sections as shown in Figure C-4. The area A_a was examined for structural adequacy since it is affected by several of the forces involved including rebound, setback, and compression due to the tension in the band when the bolts are tightened. The steel "fingers" will be supported underneath by the aluminum afterbody. Tightening of the bolts will produce a tensile stress in the band of

$$S_{2b} = 11,900 \text{ psi.}$$

The radial pressure generated by the band on the warhead is related to the stress by equation (17) such that

$$P_i = 3,010 \text{ psi.}$$

The effect of the sloped sides of the band will be to *reduce* this pressure at the base. The reduction will be the ratio of the outer and inner side dimensions of the band if the band contacts all sides equally such that

$$P'_r = P_o = 2,320 \text{ psi.}$$

The stresses in the "arm" due to this pressure vary with the radius and are related to the pressure by equations (13) and (14). In this case, the "arm" of the joint can bend like a beam and thus if energy loss through the material is neglected,

$$P_i = P_o = 2,320 \text{ psi.}$$

Equation (15) will then reduce to

$$S_r = S_t = -P = -2,320 \text{ psi.}$$

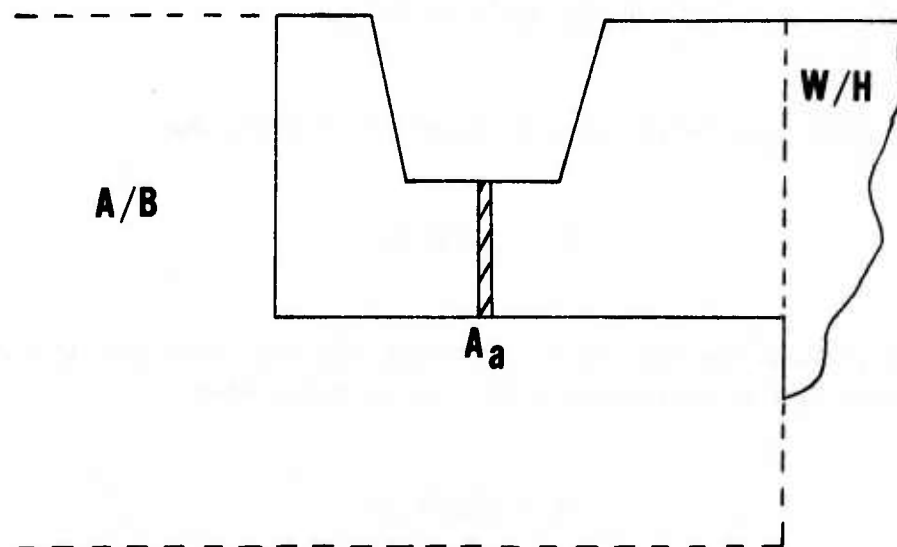


FIGURE C-4

Cross Section of Joined Bodies Under Ring

During setback, the arms will be in compression due to the transmitted load from the bearing surface on the end of the arms. The force transmitted to one of the four arms is found by finding the ratio of the bearing surface of one arm to the total bearing area and multiplying by the total setback force. In this case, the bearing area for one arm is

$$a_b = 3.68 \text{ in.}^2$$

and the total bearing area is

$$A_b = 41.95 \text{ in.}^2 .$$

$$\text{Such that } f_s = \frac{a_b}{A_b} F_s = 136,500 \text{ lb.}$$

Since

$$A_a = 4.795 \text{ in.}^2$$

then

$$S_{1s}(\text{at } A_a) = 28,500 \text{ psi} .$$

The energy yield stress theory stated as equation (12) reveals that

$$S = 30,800 \text{ psi} .$$

During rebound, the area will be in tension. However, since area $A_a = A_{sw/H}$, the stress levels will be approximately the same as before where

$$S = 27,280 \text{ psi}$$

or

$$SF = 5.50 .$$

The design appears to be particularly susceptible to thermal effects. In examining these effects, it was assumed that the coefficient of expansion, α , remained constant in the temperature range from -65° to $+160^{\circ}\text{F}$. For aluminum, $\alpha = 1.31 \times 10^{-5}$ in./deg and for steel, $\alpha = .56 \times 10^{-5}$ in./deg. The dimensions examined were taken at the basic 7-in. diameter as shown in Figure C-3. Assuming a standard temperature of 60°F , cooling of the interface area to -65°F will result in the following contractions (for $\Delta X = \alpha X_o \Delta t$):

$$\Delta X_1 = .00081 \text{ in.}$$

$$\Delta X_2 = .00132 \text{ in.}$$

$$\Delta X_3 = .00120 \text{ in.}$$

$$\Delta X_4 = .00035 \text{ in.}$$

$$\Delta X_5 = .00057 \text{ in.}$$

$$\Delta X_6 = .00052 \text{ in.}$$

$$\Delta X_7 = .00057 \text{ in.}$$

Differences between ΔX_1 and ΔX_4 , and ΔX_2 and ΔX_7 (see Figure C-5) will create compressive and shearing stresses in the band which are from equation (20)

$$S_{s_t}, S_{l_t} = -16,375 \text{ psi (each case)}$$

or a total stress

$$S = -32,750 \text{ psi .}$$

This compressive stress will also induce hoop tension stress in the band of [equation (17)]

$$S_2 = 21,800 \text{ psi}$$

which corresponds to a tensile force of

$$F_T = 18,200 \text{ lb.}$$

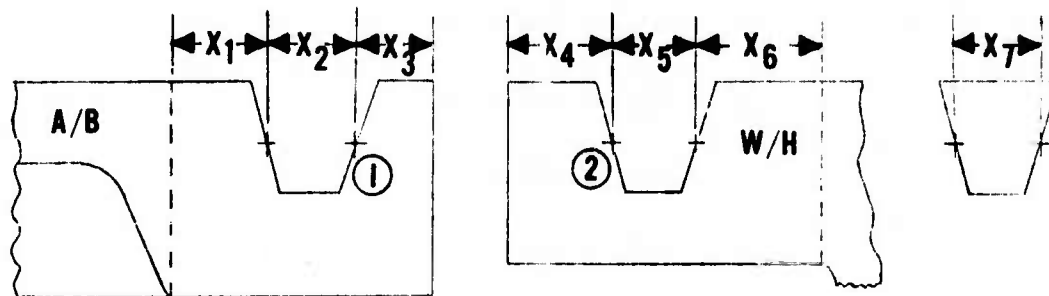


FIGURE C-5

Dimensions for Thermal Considerations

The band will experience a shear stress as a result of S_{1t} , and if the force is assumed evenly distributed over the side of the band such that the band is in double shear,

$$S_s = 24,200 \text{ psi.}$$

Comparison of F_T to the strength of the bolt indicates that the pretension in the bolts would need to be restricted to 7,000 lb.

During the heating cycle during which the temperature is elevated from 60° to 165°F, the following free expansions would result:

$$\Delta X_1 = .00069 \text{ in.}$$

$$\Delta X_2 = .00111 \text{ in.}$$

$$\Delta X_3 = .00102 \text{ in.}$$

$$\Delta X_4 = .00029 \text{ in.}$$

$$\Delta X_5 = .00048 \text{ in.}$$

$$\Delta X_6 = .00044 \text{ in.}$$

$$\Delta X_7 = .00048 \text{ in.}$$

The difference between the expansions ΔX_2 , ΔX_3 , ΔX_5 , and ΔX_6 will induce shear stresses because of restricted expansion. Again from equation (20)

$$S_{1_t} = 13,755 \text{ psi.}$$

For an evenly distributed force (for double shear), the shear stress will be

$$S_s = 17,000 \text{ psi}$$

and

$$SF = 2.59.$$

Having examined the "inverted" band joint for possible modes of failure under the given static loads, it was found that the aluminum afterbody rim would experience a shear stress during rebound of

$$S_s = 18,430 \text{ psi,}$$

which is considerably below the shear strength of the aluminum for which

$$\sigma_s = 44,000 \text{ psi.}$$

It was also observed that due to the thermal effects within the required temperature range, the initial tension on the .5-in. bolts would have to be limited to 7,000 lb or 700 lb-in. torque.

APPENDIX D

**STRUCTURAL ANALYSIS OF THE
PRESS-FIT JOINT DESIGNS**

STRUCTURAL ANALYSIS OF THE PRESS-FIT JOINT DESIGNS

It was proposed that a press-fit concept be studied for use on both the warhead/G&C and warhead/afterbody designs. The following is an analysis made on the preliminary designs for these interfaces as shown in Figures D-1 and D-2.

The basic assumptions are the same as in previous analyses but a slightly different approach is necessary, particularly in regard to the possible presence of knurling. For the knurling, it is assumed that the teeth are located on the steel warhead body and that the teeth completely embed themselves in the aluminum afterbody and G&C sections. The number of teeth will be assumed to be 14 teeth/in. resulting in a total of knurling teeth

$$\begin{aligned} N &= 2\pi rn \\ &= 305 \text{ teeth (for } r = 3.46 \text{ in.)} \end{aligned}$$

The knurling teeth shear area for a root thickness, t , and a length, L , is

$$A_s = NtL. \quad (D-1)$$

In a press-fit analysis, the displacements of the cylinders at the point of contact is of utmost interest. If for two cylinders, the internal radius of the outer cylinder in the unstressed condition is smaller than the external radius of the inner cylinder, assembly will produce a pressure, P , between the cylinders. The deformations which result are symmetrical with respect to the axis of the cylinders and consist of radial displacements of points in the wall which vary with the radius. Summation of the displacements would equal the difference between the aforementioned radii, otherwise known as the radial interference, δ . Reference (4) indicates that the displacement of any point in the wall of *either* cylinder is (see Figure D-3)

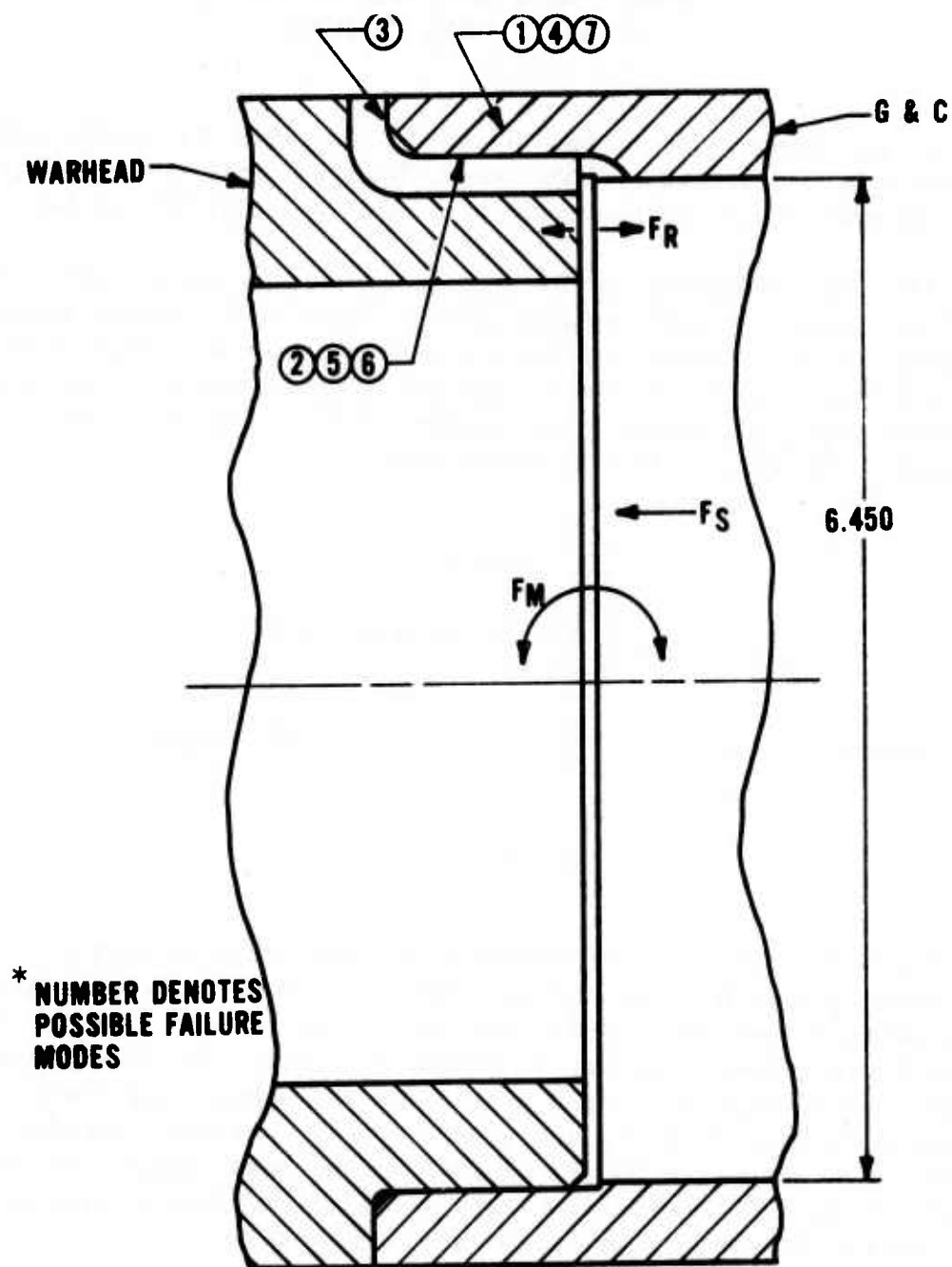


FIGURE D-1

Press Fit for the Warhead/G&C Interface Design

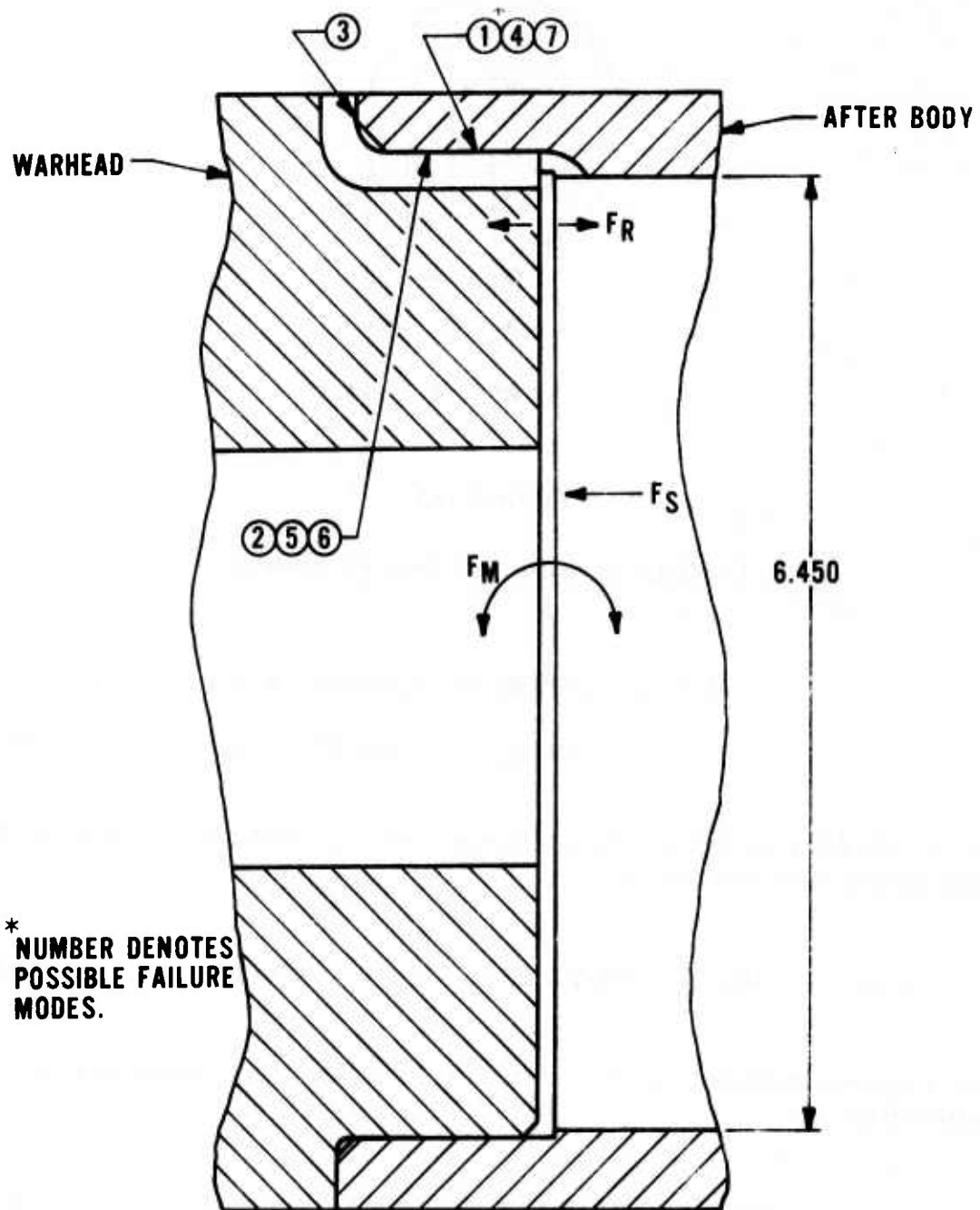


FIGURE D-2

Press Fit for the Warhead/Afterbody Interface Design

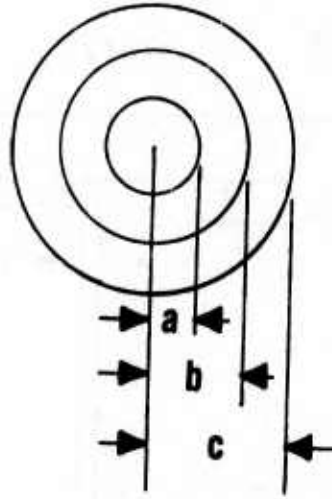


FIGURE D-3

Concentric Cylinders in a Press-Fit Analysis

$$U = [(1 + \mu)(R_i^2 R_o^2)(P_i - P_o)/(E(R_o^2 - R_i^2))]r + [(1 + \mu)(R_i^2 R_o^2)(P_i - P_o)/(Er(R_o^2 - R_i^2))] \quad (D-2)$$

For a cylinder subjected to internal pressure only, the radial displacement at the inner surface, from Equation (D-2), is

$$(U)_{r=R_i} = [R_i P_i / E_1] [(R_o^2 + R_i^2) / (R_o^2 - R_i^2) + \mu]. \quad (D-3)$$

For a cylinder subjected to external pressure only, the radial displacement at the outer surface is

$$(U)_{r=R_o} = -[R_o P_o / E_2] [(R_o^2 + R_i^2) / (R_o^2 - R_i^2) - \mu]. \quad (D-4)$$

Assuming that the increase in the inner radius of the outer cylinder plus the decrease in the outer radius of the inner cylinder equals δ , or

$$\begin{aligned}\delta &= R_{o_2} - R_{i_1} \\ &= U_{o_{r=R_c}} - U_{i_{r=R_c}}\end{aligned}$$

and substituting for U from equations (D-3) and (D-4), solving for the pressure, P , gives

$$P = E_1 E_2 \delta / [R_c (\gamma E_2 + \beta E_1)] \quad (D-5)$$

where

$$\begin{aligned}\gamma &= (R_{o_1}^2 + R_c^2) / (R_{o_1}^2 - R_c^2) + \mu_1, \\ \beta &= (R_c^2 + R_{i_2}^2) / (R_c^2 - R_{i_2}^2) - \mu_2, \text{ and} \\ R_c &= R_{o_2} - \delta [(\sigma_1 / (\sigma_1 + \sigma_2))].\end{aligned}$$

Once the pressure, P , has been obtained, equations (13)* and (14) will give the stresses in the cylinders. The inner surface of the outer cylinder also experiences a shearing stress whose maximum is

$$S_s = PR_{o_1}^2 / (R_{o_1}^2 - R_c^2). \quad (D-6)$$

Of possible importance are the radial displacements due to the spin of the projectile which could result in a loosening of the fit. Reference (4) derives the differential equation describing the displacements of a rotating disc of uniform thickness, with a hole at the center and no forces on the edges, as

* See main text of report for equations.

$$\frac{d^2 u}{dr^2} + \frac{1}{r} \frac{du}{dr} - \frac{u}{r^2} + (1 - \mu^2) \frac{\gamma \omega^2 r}{gE} = 0. \quad (a)$$

The general solution for equation (a) is

$$U = -N \frac{r^3}{8} + C_1 r + \frac{C_2}{r} \quad (b)$$

where

$$N = (1 - \mu^2) \frac{\rho \omega^2}{gE}.$$

The constants C_1 and C_2 are determined from the boundary conditions and are

$$C_1 = \frac{3 + \mu}{8(1 + \mu)} (a^2 + b^2) N$$

and

$$C_2 = \frac{3 + \mu}{8(1 - \mu)} a^2 b^2 N.$$

Substituting in equation (b) yields the general equation

$$U = -N \frac{r^3}{8} + \left[\frac{3 + \mu}{8(1 + \mu)} [a^2 + b^2] N r + \left[\frac{3 + \mu}{8(1 - \mu)} a^2 b^2 \right] \frac{N}{r} \right] \quad (D-7)$$

where

μ = Poisson's ratio,
 a, b = the inner and outer radius of the cylinder, respectively, and
 r = radius at which the displacements are desired.

For the analysis, it was assumed that the materials at the joints were steel for the warhead sections with

$$E = 30.0 \times 10^6 \text{ psi}$$

and

$$\mu = .292$$

and aluminum for other housing sections with

$$E = 10.4 \times 10^6 \text{ psi}$$

and

$$\mu = .333.$$

A study of the designs (Figures D-1 and D-2) reveals the following expected failure modes which apply to both interfaces:

1. Failure of outer cylinder from hoop stress induced by press fitting;
2. Loosening of the fit due to spin;
3. Yielding in bearing;
4. Failure of the outer cylinder during setback ;
5. Failure of joint due to torsion;
6. Failure of joint during rebound;
7. Failure from increased stresses due to thermal effects.

Warhead/G&C Interface

The analysis will proceed first with the warhead/G&C interface as shown in Figure D-1. The major quasi-static forces involved are as previously calculated for this location:

$$F_S = 348,000 \text{ lb}$$

$$F_R = 87,000 \text{ lb}$$

$$F_M = 22,200 \text{ lb}$$

Two of the other major forces, F_T and F_W , change slightly due to geometry. Thus for an O.D. = 7.95 in. and an I.D. = 5.45 in.

$$F_{T_{\text{slip}}} = 6,530 \text{ lb}$$

(for which the Torque $T = 25,900 \text{ in-lb}$).

Also, $F_w = 6,840 \text{ lb}$ where F_w has been adjusted to allow for the difference in densities of the two materials.

Referring to Figure B-3, let $r_a = 2.725 \text{ in.}$, $R_c = r_b = 3.453 \text{ in.}$ and $r_c = 3.975 \text{ in.}$ Then, if no material is moved (shaved) during assembly and $\delta = .010 \text{ in.}$, equation (19) shows that

$$P_r = 3,394 \text{ psi.}$$

The hoop stress in the cylinders as a result of this pressure is found from equation (13) such that for the outer aluminum cylinder at $r = R_c$,

$$S_2' = 24,270 \text{ psi}$$

while for the inner cylinder the maximum stresses are at $r = r_a$ where

$$S_{2(\text{inner})} = -18,000 \text{ psi.}$$

Thus for failure mode (1), a safety factor exists of

$$SF = 2.97.$$

Spin imparted to the projectile by the rifling will create inertial forces in the walls of the cylinders. These forces in turn will result in radial displacements of walls which because of the differences in the densities of the materials and the geometries will not be equal. The overall effect will be to change the amount of interference, δ , between the cylinders. This effect is found from equation (D-7) for $r = R_c$, such that

$$U_{i=r=R_c} = .000017 \text{ in.}$$

and

$$U_{o=r=R_c} = .000032 \text{ in.}$$

indicating that the total change in the overall interference is only

$$\Delta\delta = U_o - U_i = .000015 \text{ in. ,}$$

which is insignificant compared to δ and will not be considered further.

The bearing area of the G&C housing was checked for adequacy. The area was found to be 12.25 in.². Thus for $F = F_s + F_m$,

$$S_1 = -30,200 \text{ psi}$$

and

$$SF = 3.57 .$$

The compressive stress, S_1 , will be present throughout the outer cylinder in the joint area during setback except for variations in the bending moment.

Neglecting the small increase in the radial and hoop stresses due to setback, the major stresses present on an element of material on the inside surface of the outer cylinder are

$$S_1 = -30,200 \text{ psi ,}$$

$$S_2 = S_2 + S_w = 32,410 \text{ psi ,}$$

$$S_3 = -3,394 \text{ psi ,}$$

$$S_5 = 13,831 \text{ , and}$$

$$S_{S_t} \text{ (torsion)} = 533 \text{ psi}$$

where S_S is a shearing stress obtained from equation (D-6). Stress concentrations of the axial stress will occur on the inside surface due to the sharp corner at the end of the 6.900-in.-diameter countersink shown in Figure D-1. The "strength reduction factor", K_f , is from References (4) and (5), approximately

$$\begin{aligned} K_f &= 1 + q(K_t - 1) \\ &= 1.39 \end{aligned}$$

for q , the notch sensitivity factor = .41 and K_t , the theoretical stress concentration factor = 1.95. The stress concentrations will occur for both the axial, bending and torsional loads, whereas radial and hoop loads will not be affected. The strength reduction factors are for torsion

$$K_{f_t} = 1.36$$

and for bending

$$K_{f_b} = 1.39 .$$

Applying these factors to the major stresses above yields

$$S'_1 = -42,000 \text{ psi} ,$$

$$S'_2 = 32,410 \text{ psi} ,$$

$$S'_3 = -3,394 \text{ psi} ,$$

$$S_S = 13,831 \text{ psi} , \text{ and}$$

$$S_S \text{ (torsion)} = 740 \text{ psi} .$$

The principle stresses are then [equation (11)]

$$S_1 = -42,000 \text{ psi,}$$

$$S_2 = 37,140 \text{ psi, and}$$

$$S_3 = -8,115 \text{ psi.}$$

which, from equation (12), contribute to the total energy or working stress such that

$$S = 68,770 \text{ psi}$$

and

$$SF = 1.05 .$$

The torsional holding ability of the joint is of special concern for a press-fit joint. If the joint is without knurling, the torsional holding force is related to the coefficient of friction such that

$$T = 2\pi L r_b^2 (P_r) f \quad (D-8)$$

where

L = length of joint = 1.4 in., and

f = friction coefficient = .1 (min), .33 (max).

Thus for $P_r = 3,394$ psi,

$$T_{\min} = 35,500 \text{ lb-in.}$$

Since the maximum total actual torque, $T \approx 25,900$ lb-in., rotation would be unlikely. The effect of knurling was studied, however, to determine the added ability to resist rotation. Under a torsional force, mentioned above, the knurling teeth will be in shear and the shear area [equation (D-1)],

$$A_s = 30.3 \text{ in.}^2, \text{ where } t = .071 \text{ in.}$$

The yield strength to consider is that of the forward body in shear where

$$\tau_y = 44,000 \text{ psi.}$$

Thus the total force needed to fail the joint in shear is

$$F = A_s$$

$$= 1,330,000 \text{ lb}$$

and

$$SF = 51$$

The holding ability during rebound is dependent on several factors including the amount of material displaced during assembly, which effects the radial pressure, P_r , the depth of the knurling, and the coefficient of friction. Assuming that there is no knurling, and perfect concentricity, then the contact area is

$$A_c = A_s = 30.3 \text{ in.}$$

which results in a normal force from P_r of

$$F_N = 102,800 \text{ lb.}$$

Reference (6) indicates that the minimum static frictional coefficient for aluminum on steel and a dry surface is

$$f = .33,$$

thus the estimated total frictional force resisting rebound is

$$F_f = 33,900 \text{ lb.}$$

It can be seen that the introduction of knurling, with the same radial interference, results in a decrease in the radial pressure, P_r , and the normal force, F_N . Thus the total frictional force resisting rebound, F_f , will be less for a knurled surface than for an unknurled surface. Experimental results verify this conclusion, but due to the large number of variables involved, analytical results are virtually unobtainable and will not be attempted here.

In an attempt to establish a total interference needed to provide enough holding ability during rebound to avoid slippage, it was established that the minimum radial pressure, P_r , that would be needed to withstand rebound would be

$$\begin{aligned} P_r &= \frac{F_R}{fA_c} \\ &= 8,800 \text{ psi (min).} \end{aligned}$$

This would imply that the radial interference would have to be [equation (19)]

$$\delta = .026 \text{ in.}$$

Not only is this restrictive but the radial pressure of

$$P_r = 9,000 \text{ psi}$$

would mean that a hoop tension of

$$S_2 = 64,000 \text{ psi}$$

would exist in the outer collar. If the condition of a diametrical interference of .052 in. were met, assuming no material is shoved or moved when the two sections are assembled, the force required to join the sections would be an estimated 600,000 lb.

The effects of temperature variations over the range -65°F to $+160^{\circ}\text{F}$ are shown below. Cooling from 59°F (standard atmospheric temperature) to -65°F results in a tightening of the joint by

$$\Delta\delta = .0032 \text{ in.}$$

due to the difference in the radial contractions which, when hindered, would increase the radial pressure by [equation (19)]

$$\Delta P_r = 1,060 \text{ psi.}$$

Thus

$$S_2 = 7,580 \text{ psi,}$$

which would result in a total hoop stress during firing of

$$\begin{aligned} S_2 &= S_2' + \Delta S_2 + S_w \\ &= 40,000 \text{ psi.} \end{aligned}$$

Heating of the assembly from 59°F to 160°F results in a loosening of the joint by

$$\Delta\delta = -.0026 \text{ in.}$$

or

$$\Delta P_r = -860 \text{ psi.}$$

In summary, it was found that without the effects of temperature variations, the total energy (working) stress realized at the inner surface of the outer aluminum cylinder during setback is

$$S = 68,770 \text{ psi ,}$$

including all stress concentrations for a safety factor

$$SF = 1.05 .$$

However, if the temperature after assembly is lowered to -65°F , an increase in the radial pressure occurs resulting in an increase in the hoop tension during firing to a possible

$$S_2 = 40,000 \text{ psi ,}$$

which indicates a maximum working stress of

$$S = 75,000 \text{ psi .}$$

Permanent plastic deformations in extremely localized areas are possible for this stress level.

Calculations also show that the radial interference, δ , specified is inadequate for holding during rebound with or without knurling. It was estimated that a minimum total radial interference of .026 in. was necessary to hold during rebound (without knurling).

Afterbody/Warhead Interface

Looking now at the afterbody/warhead interface, a similar method of analysis will be used as in the previous interface. The major forces involved at this location have also been previously calculated and are

$$F_S = 1,560,000 \text{ lb.}$$

$$F_R = 110,000 \text{ lb.}$$

and

$$F_m = 20,700 \text{ lb (for 156 g sideload).}$$

For the design as shown in Figure D-2 and the condition that $\omega_p / \omega_B = .149$, the force due to angular acceleration is

$$F_T = 31,500 \text{ lb.}$$

Also the maximum hoop tension due to spin is [equation (7)]

$$S_w = 6,115 \text{ psi.}$$

Once pressed together, a pressure will exist between the two cylinders. For an outer radius, $R_o = 3.975 \text{ in.}$, an inner radius (of inner cylinder), $R_i = 1.50 \text{ in.}$, and a final contact radius, $R_c = 3.219 \text{ in.}$; a radial interference of $\delta = .010 \text{ in.}$ results in

$$P_r = 10,800 \text{ psi}$$

using steel properties for the outer cylinder and aluminum properties for the inner. The hoop stress in the outer cylinder as a result of this pressure is [equation (13)]

$$S_2' = 51,960 \text{ psi}$$

while for the inner cylinder, the maximum stress is

$$S_2(\text{inner}) = -27,600 \text{ psi.}$$

These would indicate that a minimum safety factor of

$$SF = 2.42$$

exists for failure mode (1).

The effect of spin on the interference, δ , is neglected. During setback, the outer steel cylinder will be in bearing on the body of the afterbody. The bearing area, based on the basic dimensions, is

$$A = 14.49 \text{ in.}^2$$

resulting in a compressive stress (including side load) of

$$S_1 = 109,000 \text{ psi}$$

and

$$SF = 1.13.$$

Elsewhere in the collar, S_1 will reduce slightly due to an increase in the area to

$$S_1 = 92,400 \text{ psi}$$

except for stress concentrations due to sharp corners and small radii. For q , the notch sensitivity factor, equal to .83 for tempered steels, and a theoretical stress concentration factor of 1.95, the strength reduction factor is

$$K_f = 1.79.$$

Applying this factor to axial and torsional stresses present in the steel outer cylinder during setback results in the following plane stresses:

$$S_1' = 165,400 \text{ psi}$$

$$S_2' = 58,000 \text{ psi}$$

$$S_3' = -10,800 \text{ psi}$$

$$S_s = 31,400 \text{ psi}$$

$$S_{s_t} = 3,290 \text{ psi}$$

The principle stresses are [equation (11)]

$$S_1 = 165,500 \text{ psi},$$

and

$$S_2 = 70,200 \text{ psi},$$

$$S_3 = -23,000 \text{ psi},$$

which contribute to a total working stress of

$$S = 163,000 \text{ psi}.$$

Since S is greater than the strength of the steel by roughly a factor of 1.3, yielding appears likely around the sharp inside corners. Elsewhere in the outer ring the Von Mises stress remains at the relatively safe level of

$$S = 106,000 \text{ psi}.$$

In the inner cylinder forward of the bearing surface, the compressive setback stress is near zero. Other stresses reach a maximum at the inner radius such that

$$S = 21,500 \text{ psi}.$$

During torsion, the knurling teeth will experience a shear stress as before. Assuming all teeth are in contact simultaneously, the total shear area is

$$A = 15.2 \text{ in.}^2,$$

thus resulting in a shear stress due to torsion

$$S_s = 2,100 \text{ psi}$$

and

$$SF = 21.2.$$

The ability of a press fit to hold during rebound will be examined once again by assuming no knurling since it is felt that this represents the *best* case. Assuming perfect concentricity of the two cylinders, the contact area is 28.3 in.^2 . Thus if the normal or radial pressure, $P_r = 10,800 \text{ psi}$, the total normal force is

$$F_N = 305,600 \text{ lb.}$$

Using the minimum expected friction coefficient $f = .33$, the estimated frictional force resisting rebound is

$$F_f = 100,900 \text{ lb.}$$

This falling short of the 130,700 lb required, the required radial pressure and interference can be calculated as before. Relating the pressure, P_r , to the normal force shows that to obtain F_f ,

$$P_r = 13,990 \text{ psi}$$

and

$$\delta = .013 \text{ in. (minimum) .}$$

Several restrictions arise from these requirements, the major problems being; first, that the shear stress resulting on the inner surface of the inner surface of the steel cylinder as a result of P_r is

$$S_s = 67,150 \text{ psi ,}$$

indicating possible yielding of the material, and second, the force estimated to press together two cylinders of diametrical interference of .077 in. is an estimated 300,000 lb.

The effects of maximum variations in the temperature over the range of -65°F to $+160^\circ\text{F}$ are shown below. Cooling of the joint area from $+59^\circ\text{F}$ to -65°F results in a loosening of the joint radially of

$$\Delta\delta = -.0023 \text{ in.}$$

which would decrease the radial pressure [equation (19)] by

$$\Delta P_r = -2,500 \text{ psi .}$$

The reduction in the hoop tension is consequently

$$\Delta S_2 = -12,000 \text{ psi ,}$$

resulting in a total hoop stress in the outer cylinder during setback of

$$S_2 = 46,100 \text{ psi.}$$

Heating of the assembly from 59°F to 160°F results in a tightening of the fit by

$$\Delta\delta = .0019 \text{ in.}$$

or

$$\Delta P_r = 2,050 \text{ psi.}$$

In summary, it is observed that there exist two major weaknesses in the press fit design for the afterbody/warhead similar to those found in the forward joint. During setback, the working stress occurring in the outer cylinder near the inner surface is

$$S = 106,000 \text{ psi}$$

if stress concentrations are not taken into account and is

$$S = 163,000 \text{ psi}$$

if the stress concentrations are considered, indicating that some areas will likely experience some plastic deformations. Also indicated was the inadequacy of the interference, δ , to develop sufficient pressure between the two cylinders to prevent slippage during rebound. It was estimated that a minimum radial interference of .013 in. was necessary to withstand the 130,700 lb total rebound and moment forces. In additions, the combined effects of knurling and material displacement will necessitate an estimated 50-percent increase in the radial interference such that $\delta = .019 \text{ in.}$ in order to obtain P_r .

APPENDIX E

**STRUCTURAL ANALYSIS OF THE
FOUR-BOLT JOINT DESIGN**

STRUCTURAL ANALYSIS OF THE FOUR-BOLT JOINT DESIGN

The design as analyzed is shown in Figure E-1. The forces applicable for the afterbody/warhead joint location as calculated previously in Appendix A are:

$$F_S = 1,560,000 \text{ lb}$$

$$F_R = 110,000 \text{ lb}$$

$$F_m = 20,700 \text{ lb}$$

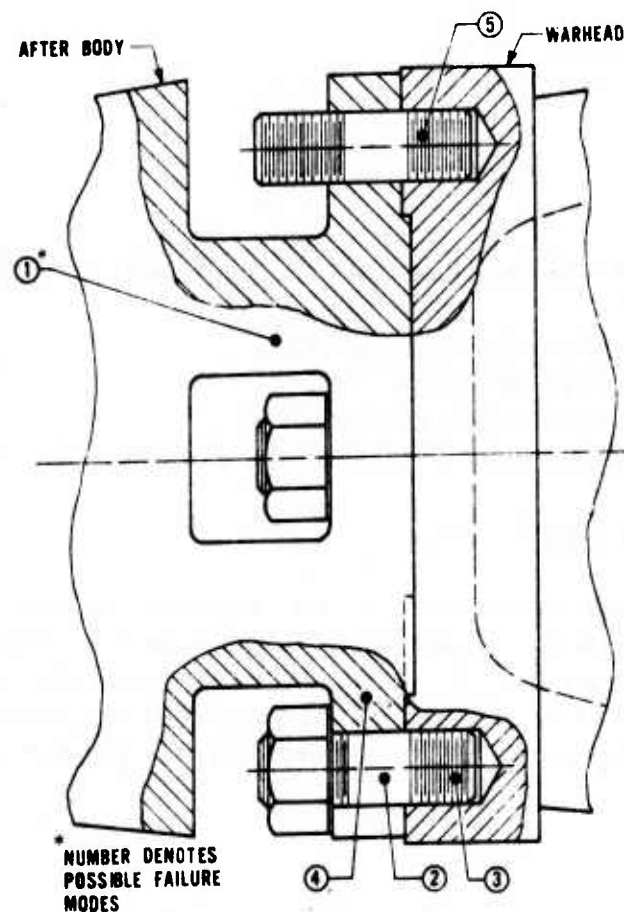


FIGURE E-1

Four-Bolt Joint Design

In this design it will be assumed that the 3-in.-diameter hole in the center of the afterbody will not be present as in previous designs. Based on the major assumptions outlined in the text, the tangential force is [equation (3)*]

$$F_T = 33,400 \text{ lb.}$$

Also the maximum hoop stress in the wall due to spin is [from equation (7)] for the afterbody

$$S_w = 655 \text{ psi}$$

and for the steel warhead

$$S_w = 1,836 \text{ psi.}$$

An examination of the design indicates the following possible modes of failure:

1. Failure of the afterbody during setback;
2. Failure of the bolts in tension;
3. Failure of the threads during rebound;
4. Failure of the afterbody in shear in the area under the stud nuts during rebound;
5. Failure of the bolts in shear.

During the forward acceleration of the projectile, setback stress will be transmitted totally through the bearing surface between the two units. The afterbody as shown has an outer diameter of 7.75 in. Thus the bearing area, after removal of the four bolt holes, is 45.41 in.². Due to the location of the joint rearward of the rotating band, the force acting on the bearing area is a function of the chamber pressure, P, such that

* Refer to main text for equations.

$$F = P(A - A_1)$$

where A is the maximum cross-sectional area rearward of the joint and A_1 is the area of the rotating band outside of A on which the pressure, P , acts. Thus, for $A = 46.32$ and $A_1 = 3.94 \text{ in.}^2$, then

$$F = 1,695,200 \text{ lb.}$$

The resulting bearing stress is

$$S_{1_b} = 37,330 \text{ psi.}$$

The stress, S_1 , will increase in the afterbody in the cross section including the cavities for excess to the bolts since the area will be reduced. The total reduction, including the fin slots, will bring the area down to 25.44 in.^2 , resulting in a compressive stress of

$$S_1 = 66,635 \text{ psi}$$

and

$$SF = 1.08.$$

The only effect of setback on the bolts will be to loosen or reduce the pretension in the bolts. Loosening can be prevented (some tension will always be present) if the stress under the nut due to the bolt torque is as great as the stress in the same area due to setback. Thus, if the stress is $37,330 \text{ psi}$ and the nut bearing area is 1.08 in.^2 , the pretension in each bolt must be

$$F = 40,335 \text{ lb.}$$

The torque required for an unlubricated bolt to achieve this tensile force is approximately

$$T = 505 \text{ ft-lb.}$$

Failure modes (2), (3), and (4) are dependent upon the magnitude of the force during rebound. In calculating the tension in the bolts, the tension due to the initial torque will not add to the tension due to rebound. Therefore, assuming a worst possible condition such that the moment force is applied fully on one bolt, and for a tensile area of $.373 \text{ in.}^2$, the total tensile stress in each bolt is

$$S_1 = 129,200 \text{ psi.}$$

The studs chosen have a tensile strength of

$$\sigma_y = 180,000 \text{ psi,}$$

thus

$$SF = 1.39.$$

The bearing area of the stud nut was calculated to be $.91 \text{ in.}^2$. Therefore during rebound, the bearing stress will be

$$S_{1b} = 52,970 \text{ psi}$$

and

$$SF = 2.33.$$

using a bearing strength for the aluminum, $\sigma_b = 123,000 \text{ psi}$. The length of engagement of the bolt with the warhead was examined. The minimum thread engagement required for a given load, F , is found by the following equation, which has been rearranged from the equation given by Reference (7) where

$$L_e = \frac{2FP}{\sigma_y} \left(\pi K_{n_{max}} \left[1/2n + .57735(E_{S_{min}} - K_{n_{max}}) \right] \right)^{-1}$$

where

$$p = \text{pitch} = \frac{1}{n},$$

$K_{n_{max}}$ = maximum minor diameter of internal thread,

$E_{S_{min}}$ = minimum pitch diameter of external thread, and

y = tensile yield strength of weakest material.

For the design chosen, $n = 12$ threads per inch, $K_{n_{max}} = .6820$ in. and $E_{S_{min}} = .7391$ in. which indicates a minimum thread engagement of

$$L_e = .40 \text{ in.}$$

The thread engagement chosen was 1.05 in.

therefore

$$SF = 2.63 .$$

The afterbody portion under the nut will be in shear as well as bearing. The area in shear for the .75-in.-thick section is

$$A_s = 2.04 \text{ in.}^2$$

which, for the loads given, will experience a shear stress

$$S_{s_1} = 23,600 \text{ psi}$$

and

$$SF = 1.86 .$$

The final mode of failure possible is the shearing of the bolts due to the tangential force resulting from the angular acceleration. Assuming each bolt is sharing the load equally, then for $A_s = .373 \text{ in.}^2$,

$$S_{s_b} = 22,400 \text{ psi} .$$

In summary, it was determined that the reduction in strength of the afterbody by the removal of the material for the four access holes was not critical. However, in obtaining the compressive stress, S_1 , the quasi-static force resulting from the breech chamber pressure was used which was equivalent to the setback force at the joint. Instantaneously, the force due to pressure could be 1.25 times this amount such that the stress, S_1 , may become

$$S_1 (\text{instantaneous}) = 77,000 \text{ psi} .$$

thus indicating that the stresses in this area are critical. Otherwise, the stresses in the design are well within its capabilities with the bolts themselves being the next weakest feature where $SF = 1.39$ at design loads.

APPENDIX F
DISTRIBUTION

DISTRIBUTION

Deputy Director
DDRE (Tactical Warfare Programs)
The Pentagon
Washington, D. C. 20350

Assistant Secretary of the Navy (R&D)
The Pentagon
Washington, D. C. 20350

Center for Naval Analysis
The Pentagon
Washington, D. C. 20350

Chief of Naval Material
Department of the Navy
Washington, D. C. 20350
Attn: J. H. Huth
Code 03L
Code 0324

Chief of Naval Operations
The Pentagon
Washington, D. C. 20350
Attn: Code 90
Code 00K1
Code 098
Code 351G
Code 06

Commander
Naval Sea Systems Command
Washington, D. C. 20360
Attn: SEA-653
SEA-654
SEA-033
SEA-99
SEA-034

(2)

(3)

Commander
Naval Air Systems Command
Washington, D. C. 20360
Attn: AIR-320

Commandant of the Marine Corps
Headquarters Marine Corps
Washington, D. C. 20380
Attn: AX-2
A04F035
A03H
AX4E1

Director, Development Center
Marine Corps Development and Education
Quantico, Virginia 22134
Attn: Ground Operation Division

Commander
Naval Surface Weapons Center
White Oak Laboratory
Silver Spring, Maryland 20910
Attn: S. Hastings
Code 423

Commander
Naval Weapons Center
China Lake, California 93555
Attn: Code 457

Commanding Officer
Naval Missile Center
Point Mugu, California 93042

Commander
Naval Ship Research and Development Center
Washington, D. C. 20007

Commanding General
U. S. Army Weapons Command
Rock Island Arsenal
Rock Island, Illinois 61201
Attn: AMSEW-REV-SPA
PM-CAWS
SWERR-T-SPA

Commanding General
Picatinny Arsenal
Dover, New Jersey 07801
Attn: Code SMUPA-AD-D

Commanding General
U. S. Army Missile Command
Redstone Arsenal, Alabama 35809
Attn: AMSMI-RFS (LTC Farrington)

Commanding General
Ballistic Research Laboratories
Aberdeen Proving Ground
Aberdeen, Maryland 21005
Attn: C. H. Murphy
R. M. Marchetti

Defense Documentation Center
Cameron Station
Alexandria, Virginia 21314

(2)

Commander
Naval Ordnance Station
Indian Head, Maryland 20640
Attn: Code FS-73

Commander
Naval Avionics Facility
Indianapolis, Indiana 46218

Harry Diamond Laboratories
Conn. & Van Ness Sts., N. W.
Washington, D. C. 20425
Attn: Code AMXDOD-DAD

U. S. Army Munitions Command
Dover, New Jersey 07801
Attn: AMSMU-RE-CA

Operational Test and Evaluation Agency
Field Test Division
Ft. Belvoir, Virginia 22060
Attn: LTC R. H. Sugg

Commander
Operational Test & Evaluation Forces
U. S. Naval Base
Norfolk, Virginia 23511
Attn: CDR Hooks

Commander
HQ, TRADOC
Ft. Monroe, Virginia 23651
Attn: ATCD-CD (MAJ Ayers)

Commandant
USAFAS
Ft. Sill, Oklahoma 73503
Attn: ATSF-CTD-WC (Mr. Fred Shelton)

Commander
HQ AMC
5001 Eisenhower Ave.
Alexandria, Virginia 22304
Attn: AMCRD-W (Mr. Dick Happick)

Local:

C
D
F
G
K
T
GP
GP (Doi)
GP (Crist)
GP (Brandts)
GW
KBB (Mason) (5)

GW (Aids)

MIP-2

MIP-3

MIM

MIL

(5)

(2)

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

| REPORT DOCUMENTATION PAGE | | READ INSTRUCTIONS BEFORE COMPLETING FORM |
|---|-----------------------|--|
| 1. REPORT NUMBER TR-3082 | 2. GOVT ACCESSION NO. | 3. RECIPIENT'S CATALOG NUMBER |
| 4. TITLE (and Subtitle) A STATIC STRUCTURAL ANALYSIS OF VARIOUS MAJOR COMPONENT JOINING DESIGNS FOR THE 8-INCH GUIDED PROJECTILE | | 5. TYPE OF REPORT & PERIOD COVERED |
| | | 6. PERFORMING ORG. REPORT NUMBER |
| 7. AUTHOR(s) Lawrence A. Mason | | 8. CONTRACT OR GRANT NUMBER(s) |
| 9. PERFORMING ORGANIZATION NAME AND ADDRESS Naval Surface Weapons Center Dahlgren Laboratory Dahlgren, Va. 22448 | | 10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS |
| 11. CONTROLLING OFFICE NAME AND ADDRESS | | 12. REPORT DATE March 1975 |
| | | 13. NUMBER OF PAGES |
| 14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) | | 15. SECURITY CLASS. (of this report) UNCLASSIFIED |
| | | 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE |
| 16. DISTRIBUTION STATEMENT (of this Report) Distribution limited to U.S. Gov't agencies only; Test and Evaluation; (3-75). Other requests for this document must be referred to OIC and Assistant Commander, Naval Surface Weapons Center, Dahlgren Laboratory, Dahlgren, Virginia 22448. | | |
| 17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report) | | |
| 18. SUPPLEMENTARY NOTES | | |
| 19. KEY WORDS (Continue on reverse side if necessary and identify by block number) | | |
| 20. ABSTRACT (Continue on reverse side if necessary and identify by block number) This report contains a description of the structural analysis of several candidate joining designs for major components of the 8-inch Guided Projectile. The analyses contained herein are not intended to be rigorous in approach, but are conducted in such a manner as to cover the pertinent design details which would effect the overall structural integrity of both the | | |

DD FORM 1 JAN 73 1473

EDITION OF 1 NOV 65 IS OBSOLETE
S/N 0102-014-6601

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)

joining mechanism itself and the adjacent sections of the major components involved.

The information contained in this report includes analyses of:
(1) the three-section ring or "Marman" band concept for both the warhead/afterbody and warhead/guidance and control interfaces, (2) the two-ring inverted Marman band, (3) the press-fit approach (also for both warhead interfaces), and (4) the bolted-joint concept.

The results of these analyses indicate that the simpler bolted-joint design exhibits the greatest potential for success, although no design is completely void of potential problem areas. Recommendations are made to improve the design where possible.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)

U.S. NAVAL WEAPONS LABORATORY SCIENTIFIC, TECHNICAL and ADMINISTRATIVE PUBLICATIONS

TECHNICAL REPORTS: Scientific and technical information on the work of the Laboratory for general distribution.

TECHNICAL NOTES: Preliminary or partial scientific and technical information, or information of limited interest, for distribution within the Laboratory.

CONTRACTOR REPORTS: Information generated in connection with Laboratory contracts and released under NWL auspices.

ADMINISTRATIVE REPORTS: Administrative information on the work, plans and proposals of the Laboratory.

ADMINISTRATIVE NOTES: Preliminary or partial administrative information, or information of limited interest, for distribution within the Laboratory.

Details on the availability
of these publications
may be obtained from:

**TECHNICAL LIBRARY
U.S. NAVAL WEAPONS LABORATORY
DAHLGREN, VIRGINIA 22448**